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Analytical study of heat transfer through friction overlay of band-pad brakes

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Abstract. The paper analyses the operating conditions of band-pad brakes of drilling winch, materials and coatings for strengthening the metal elements of friction pairs. Methods of researching the thermal and stress-strain state of elements of brake friction units are considered. The problem of the development and theoretical research of the properties of new polymer materials with gradient properties for the production of friction overlays of band-pad brakes with improved operational characteristics is singled out. A mathematical model heat transfer through friction overlays of band-pad brakes of drilling winches with a variable coefficient of thermal conductivity of the material based on the thickness of the specified overlays has been developed. According to the developed mathematical model, an analytical study of the process of heat transfer through the friction overlays of band-pad with variable coefficient of thermal conductivity of their polymer material according to the thickness of the friction overlays was carried out according to different laws. It was established that the highest intensity of heat removal from the friction zone of the brakes is ensured when the thermal conductivity coefficient changes according to the law described by a second-order polynomial.
Keywords: polymer brake overlay, thermal conductivity coefficient, wear, functionally gradient material, coating, mathematical model.

1. Introduction

To increase the volume of oil, gas and coal production, it is necessary to increase the volume of well drilling, and this, in turn, requires the development of high-performance drilling equipment and tools with improved operational characteristics [1, 2, 3]. One of the ways to increase the productivity of the well drilling process is to reduce the time of lowering and lifting operations for replacing worn drill bits and increasing the length of the drill pipe column, which requires the use of a fast-acting and reliable braking system of drilling rig winches equipped with band-pad brakes.

The friction process in the metal-polymer friction pairs “metal pulley – friction polymer overlay” of the band-pad brakes is a high-energy phenomenon. To ensure stable thermodynamic conditions for the performance of the tribo assembly, the elements of its friction pairs must dissipate the generated thermal energy during braking in such a way that the material of the friction polymer overlays do not reach the maximum permissible surface temperature. This thermodynamic state of overlays allows to stabilize the process of friction and wear.

During lowering and lifting operations of drill strings with a high speed of movement, the cooling time of the elements of the friction pairs is sharply reduced. It can lead to overheating of the friction polymer overlays and their increased wear, and as a result, premature failure of the braking system of the drilling winch and the occurrence emergency situations. When developing brake structures, researchers focus on the rational choice of materials, the study of temperature and stress distribution, wear tests [4], including taking into account the electrochemical indicators of tribocorrosion [5] and tribocurrents [6], as well as the study of floor images [7].

Steel, cast iron, aluminum, titanium alloys and composite metal and non-metal polymer materials are used as materials for the manufacture of brake parts. Brake discs, drums, and pulleys are usually made from cast or rolled billets and 3D printed. During the manufacture of machine parts, considerable attention is paid to achieving accuracy [8, 9, 10] and the quality of operational surfaces [11, 12, 13], optimization of technologies [14], iincluding taking into account technological heredity [15], to ensure long-term operation of engineering products during their life cycle [16, 17].

To increase the wear resistance of the operational surfaces of steel discs and brake drums, friction hardening [18], laser processing [19] and coating are formed in various ways, for example, by vacuum-arc deposition [20], electric spark deposition [21, 22], as well as surfacing [23, 24]. Electrochemical chrome coatings [25] have high wear resistance, but are not used due to the low coefficient of friction in a pair with a polymer brake overlay.

Theoretical approaches to the development of the composition of new composite materials and strengthening technologies are proposed in [26, 27]. Researchers [28] developed a method of thermodynamic prediction of the phase composition of high-entropy alloys of transition metals for parts of friction nodes, including tungsten-free ones [29]. It is promising to use composite oxide coatings formed by plasma electrolytic oxidation in an electrolyte at high voltages [30, 31] to strengthen the operational surfaces of disks and drums made of aluminum and titanium alloys.

Polymeric materials are widely used in various friction pairs [32]. Friction linings are made by casting, pressing, as well as 3D printing, which is a prerequisite for creating functionally gradient materials for brakes.

To ensure trouble-free operation of braking systems, it is necessary to study the temperature and stress-strain state of the friction parts. The works [33, 34, 35] developed an analytical method for studying heat transfer and calculating non-stationary temperature fields in multilayered bodies of simple shapes. The works [36, 37] present a method of calculating the stress-strain state of layered structures during the application of forces, respectively, perpendicular to the surface of the layers and parallel to them and near defects [38].

Research of complex objects is carried out with the help of computer modeling. Using the finite element method [39], the force interaction in the friction pair of metallic elements – non-metallic rock was investigated without taking temperature factors into account. Researchers [40] performed a simulation of the frictional contact of a disc brake and established the nature of the temperature distribution.

Traditional friction polymer materials do not fully satisfy modern growing requirements for ensuring the stable performance of parts under extreme operating conditions of the drill winch brake. The results of tribological studies of materials with constant thermal conductivity are usually described in the literature, but there is practically no information on the behaviour of materials with a variable coefficient of thermal conductivity in friction pairs.

In paper [41], the process of wear and heating of the composite overlay with a constant coefficient of thermal conductivity was experimentally verified. The work [42] investigated the temperature distribution in the elements of a friction pair consisting of functionally gradient and homogeneous materials, but did not consider various options for changing the thermal conductivity of materials.

Thus, there is a need to develop and theoretically study the new polymer materials with gradient properties for friction overlays of band-pad brakes, as well as the creation of analytical methods for calculating and analysing the thermal state of friction brake pair elements.

This study aims to carry out an analytical study of heat transfer through the friction overlay of band-pad brakes of drilling winches with a variable coefficient of thermal conductivity of the material.

To achieve the goal, the following tasks were set:

- to develop a mathematical model of heat transfer through the friction overlays of band-pad brakes of drilling winches with a variable coefficient of thermal conductivity of the material;
- investigate the process of heat transfer through the friction overlays of the band-pad brake with a variable thermal conductivity coefficient of their polymer material and choose a rational law of thermal conductivity change.

2. Materials and Methods

2.1. Studied brake unit, materials of its friction elements

The band-pad brake of drilling winches (U2-5-5) include a metal brake pulley with side flanges (Steel 35KhNL State standard GOST 4543-71), wrapped around the bang with a set of friction polymer overlays attached to it. The run-up end of the metal band is connected to the adjusting tension screw, and the run-up end of the band is hinged to the mechanical brake control actuator.

Analytical studies of heat transfer through the friction overlays of band-pad brakes of drilling winches have been carried out (figure 1). Retinax of the A brand (FK-16L) State standard GOST 10851-94 and polymer material with variable thermal conductivity coefficient depending on the thickness of the overlay were chosen as the main material of the overlay. To increase the efficiency of heat removal from the friction zone through the friction pads, it was proposed to use a functionally gradient material with the thermal conductivity coefficient varying by thickness in the radial direction of this pad according to various dependencies (Fig. 2). The radius of the pulley was r_0 = 0.725 m, and the thickness of the polymer overlay was 0.02 m.

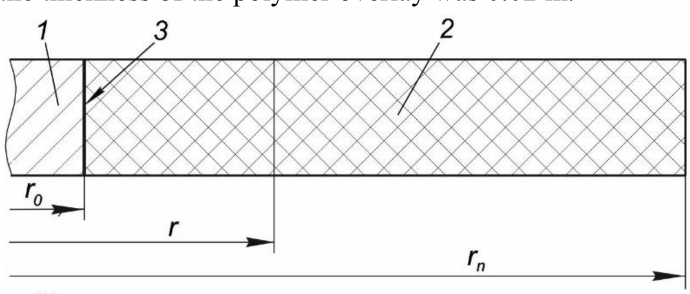

Figure 1. Calculation diagram of heat transfer study through friction overlays with variable thermal conductivity depending on the overlay thickness (in the radial direction) of the band-pad brake of drilling winches: 1 – pulley; 2 – overlay; 3 – friction zone.

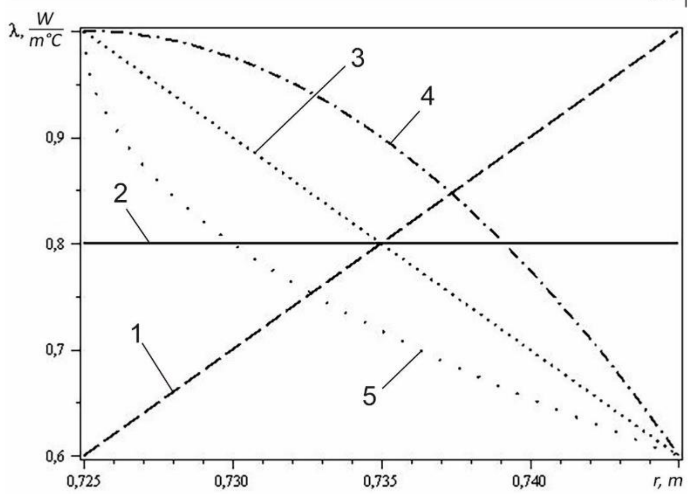

Figure 2. Laws of change in the coefficient of thermal conductivity, which varies depending on the thickness of the overlay in the radial direction:

1 – $\lambda(r) = const = 0,8 \ \frac{W}{m \cdot {}^0C}$;

2 – $\lambda(r) = 20r - 13,9$;

3 – $\lambda(r) = -20r + 15,5$;

4 – $\lambda(r) = 1 - 0,2\sqrt{-145 + 200r}$;

5 – $\lambda(r) = -1000r^2 + 1450r - 254,625$

By the time of braking, the ambient temperature is +20 °C. The surface temperature of the friction between the overlay and the disk is +1100 °C. On the other hand, there is a process of heat exchange between the overlay and the surrounding environment (we neglect the metal tape located on the outer surface of the polymer overlay).

2.2. Analytical research methodology and problem formulation
The process of heat exchange in the overlay can be described by a mathematical model, which includes the differential equation of heat conduction in a cylindrical system [43]

$$c\rho\frac{\partial t(r,\tau)}{\partial\tau}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial t(r,\tau)}{\partial r}\right), \tag{1}$$

with a system of boundary conditions of the third kind

$$\begin{cases}\alpha_0 r_0 t(r_0,\tau)-t^{[1]}(r_0,\tau)=\alpha_0 r_0\psi_0(\tau),\\ \alpha_n r_n t(r_n,\tau)+t^{[1]}(r_n,\tau)=\alpha_n r_n t_c(\tau),\end{cases} \tag{2}$$

and the initial condition as

$$t(r,0)=\varphi(r). \tag{3}$$

Here $t(r,\tau)$ – the distribution of the temperature field, $r_0<r_1<\ldots<r_n$ – is an arbitrary division of the interval $[r_0,r_n]$ of real axis into *n* parts, $t^{[1]}\overset{df}{=}r\lambda t'$ – is the quasi-derivative (the physical essence – is the heat flux density *q* multiplied by the radius *r*), *c* – the specific heat capacity, ρ – the density, α_0 – is the heat exchange coefficient between the friction zone and the inner surface of the brake overlay, α_n – the heat exchange coefficient between the outer surface of the brake overlay and the environment, $\psi_0(\tau)$ – the law of temperature change in the friction zone [44].

3. Results

To confirm the assumptions, an analytical study was conducted to determine the amount of heat that leaves the brake overlay over a certain time interval.

The amount of heat removed from the working surface of the brake overlay is determined by the formul [45]

$$Q_{surf}=2\pi r_{surf}q\tau, \tag{4}$$

where Q_{surf} – where, is the amount of heat dissipated from the working surface of the brake overlay, r_{surf} – is the coordinate of the outer side of the overlay, q – is the heat flux density at point r_{surf}, τ – time.

The heat flux density is determined as

$$q=-\lambda\frac{\partial t(r_{surf},\tau)}{\partial r}, \tag{5}$$

where $t(r_{surf},\tau)$ – is the distribution of the temperature field on the surface of the brake overlay, λ – is the thermal conductivity coefficient.

In order to determine the amount of heat that leaves the surface of the overlay, it is necessary to solve the problem (3) – (5), we will carry out according to the following scheme, which is studied in detail and described in the work [44]:

1. The solution $t(r,\tau)$ is sought by the method of reduction in the form

$$t(r,\tau)=u(r,\tau)+v(r,\tau)\,. \tag{6}$$

2. A quasi-stationary boundary value problem is solved for one of the functions (for example, for $u(r,\tau)$ the function)

$$\frac{1}{r}\frac{d}{dr}\left(r\lambda\frac{du(r,\tau)}{dr}\right)=0, \tag{7}$$

with the boundary conditions (4) for the function $u(r,\tau)$, i.e.

$$\begin{cases}\alpha_0 r_0 u(r_0,\tau)-u^{[1]}(r_0,\tau)=\alpha_0 r_0\psi_0(\tau),\\ \alpha_n r_n u(r_n,\tau)+u^{[1]}(r_n,\tau)=\alpha_n r_n\psi_n(\tau).\end{cases} \tag{8}$$

The solution structure of problem (7) – (8) is described in detail in the work [44].
On each of the intervals $[r_i,r_{i+1})$, the solution of this problem has the form

$$\mathbf{u_i}(r,\tau)=B_i(r,r_i)\cdot B(r_i,r_0)\cdot\mathbf{P_0}(\tau), \tag{9}$$

$$\mathbf{P_0}(\tau)=\left(P+Q\cdot B(r_n,r_0)\right)^{-1}\cdot\mathbf{\Gamma}(\tau)=$$

$$=\left[\begin{pmatrix}\alpha_0 r_0 & -1\\ 0 & 0\end{pmatrix}+\begin{pmatrix}0 & 0\\ \alpha_n r_n & 1\end{pmatrix}\begin{pmatrix}1 & \sum\limits_{i=0}^{n-1}\frac{\ln r_{i+1}-\ln r_i}{\lambda_i}\\ 0 & 1\end{pmatrix}\right]^{-1}\begin{pmatrix}\alpha_0 r_0\psi_0(\tau)\\ \alpha_n r_n t_c(\tau)\end{pmatrix}=$$

$$=\begin{pmatrix}\dfrac{(\alpha_n r_n\sigma_n+1)\alpha_0 r_0\psi_0(\tau)+\alpha_n r_n t_c(\tau)}{\alpha_n r_n+1+\alpha_0 r_0(\alpha_n r_n\sigma_n+1)}\\ \dfrac{\alpha_0\alpha_n r_0 r_n\left(t_c(\tau)-\psi_0(\tau)\right)}{\alpha_n r_n+1+\alpha_0 r_0(\alpha_n r_n\sigma_n+1)}\end{pmatrix}.$$

The expression (9) allows writing the solution of the problem on the interval $[r_0,r_n]$ using characteristic functions θ_i in the form as

$$\mathbf{u}(r,\tau)=\sum_{i=0}^{n-1}\mathbf{u_i}(r,\tau)\theta_i. \tag{10}$$

where $\theta_i(r)=\begin{cases}1, r\in[r_i,r_{i+1}),\\ 0, r\notin[r_i,r_{i+1}), i=\overline{0,n-1}.\end{cases}$

3. We get a mixed heterogeneous problem for the function $v(r,\tau)$

$$c\rho\frac{\partial v(r,\tau)}{\partial\tau}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial v(r,\tau)}{\partial r}\right)-c\rho\frac{\partial u(r,\tau)}{\partial\tau}. \tag{11}$$

with zero boundary conditions for the function $v(r,\tau)$,

$$\begin{cases}\alpha_0 r_0 v(r_0,\tau)-v^{[1]}(r_0,\tau)=0,\\ \alpha_n r_n v(r_n,\tau)+v^{[1]}(r_n,\tau)=0,\end{cases} \tag{12}$$

and the initial condition

$$v(r,0) = f(r) = \varphi(r) - u(r,0). \tag{13}$$

The solution of the mixed problem (11) – (13) is obtained in the form of a series [44]

$$v(r,\tau) = \sum_{k=1}^{\infty}\left[f_k \cdot e^{-\omega_k \tau} - \int_0^{\tau} e^{-\omega_k(\tau - s)} u_k(s)\,ds \right] \cdot R_k(r,\omega_k) = \sum_{i=0}^{n-1} v_i(r,\tau)\cdot\theta_i, \tag{14}$$

where, ω_k – the roots of the characteristic equation of the eigenvalue problem, $R_k(r,\omega_k)$ – the eigenfunctions of the corresponding eigenvalue problem, f_k – the Fourier coefficients of the expansion of the initial condition (13), $u_k(s)$ – the Fourier coefficients of the expansion of the function $\frac{\partial u(r,\tau)}{\partial \tau}$ eigenfunctions.

Considering the definition (6) and formulas (10) and (14), we obtain the solution of problem (3) – (5) in the form

$$t(r,\tau) = \sum_{i=0}^{n-1}\left[u_i(r,\tau) + v_i(r,\tau) \right]\cdot\theta_i. \tag{15}$$

Using the above-described method, the amount of heat that leaves the surface of the brake overlay was investigated depending on the law of change in the thermal conductivity coefficient (figure 3).

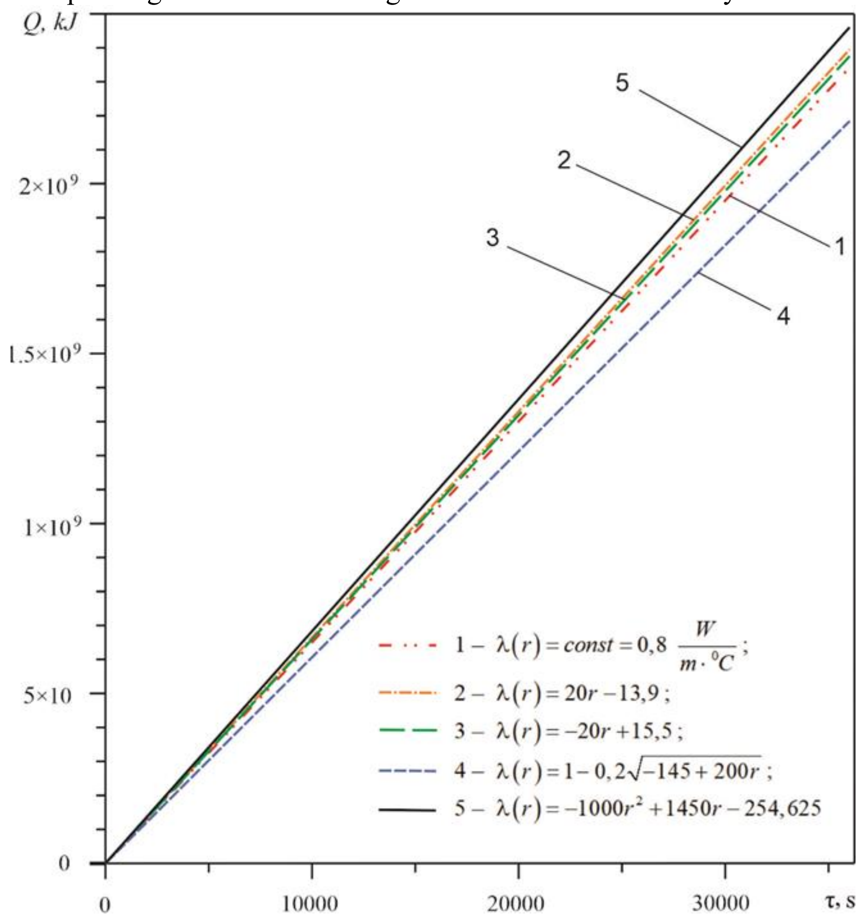

Figure 3. The amount of heat dissipated through the body of the friction overlay, as a function of time under different laws of the thermal conductivity coefficient change (1 – 5).

Figure 3 shows the dependence of the amount of heat dissipated through the body of the polymer overlay a certain time interval under different laws of change in the thermal conductivity coefficient depending on the thickness of the overlay:

– $\lambda(r) = const = 0,8 \; \frac{W}{m \cdot {}^0C}$ (figure 3, line 1);
– $\lambda(r) = 20r - 13,9$ (figure 3, line 2);
– $\lambda(r) = -20r + 15,5$ (figure 3, line 3);
– $\lambda(r) = 1 - 0,2\sqrt{-145 + 200r}$ (figure 3, line 4);
– $\lambda(r) = -1000r^2 + 1450r - 254,625$ (figure 3, line 5).

The analysis of the calculations shows that the coefficient of thermal conductivity, at which the least amount of heat is removed from the friction zone through the body of the polymer overlay, varies according to the law $\lambda(r) = 1 - 0,2\sqrt{-145 + 200r}$, and the most according to the law $\lambda(r) = -1000r^2 + 1450r - 254,625$.

Thus, the developed mathematical model of the process of heat transfer through frictional polymer overlay s (with a variable coefficient of thermal conductivity depending on its thickness) of band-pad brakes of drilling winches allow the use of materials with gradient properties during the design and manufacture of brake overlay s. This will allow provide modern requirements for achieving stable performance of the parts of the brake unit of the drilling winch under extreme operating conditions.

4. Conclusions

The mathematical model of heat transfer through friction overlay s made of polymer material with the variable thickness in the radial direction of the overlays thermal conductivity coefficient, of a band-pad brake of a drilling winch, has been developed. This model makes it possible to analytically estimate the amount of heat that is released during the corresponding time interval through the overlay, depending on the law of change of the thermal conductivity coefficient.

Five different laws (according to the type of overlay) of the variation of the coefficient of thermal conductivity by thickness in the radial direction of the friction overlay were studied. It was established that the most heat leaves the surface of the polymer overlay when the coefficient of thermal conductivity changes according to the law described by the dependence $\lambda(r) = -1000r^2 + 1450r - 254,625$.

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