The object of the study is a circular saw blade. An annular plate of constant thickness with a free outer contour and a fixed inner contour was taken as the calculation scheme of the saw blade. The real conditions for fixing the internal circuit correspond to the elastic fixing of the saw blade with clamping flanges on the shaft of the machine. For the accepted calculation scheme of the circular saw, the dynamic model was a fourth-order nonlinear differential equation of the transverse oscillations of the annular plate with the corresponding boundary conditions. The rotation of the circular saw was taken into account in the dynamic model due to the radial force in the middle surface of the ring plate. This force arises as a result of the action of centrifugal forces during the rotation of the saw blade. The solution to the fourth-order nonlinear differential equation was constructed using the Bubnov-Galyorkin numerical method. The boundary conditions for constructing the solution were as follows: the outer contour of the saw disk was considered free; the inner contour of the saw disk – elastically fixed with a certain stiffness coefficient.

The solution was implemented in the Maple 15 mathematical environment in the form of a developed program. According to the obtained frequency equation, the values of cyclic and natural frequencies of transverse oscillations of circular saw disks of different thicknesses with the same radius of the inner contour and three values of the radii of the outer contour were determined: 150 mm, 200 m, and 250 mm. The effect of the rigidity of the internal contour fixing and the angular speed of rotation of the saw blade on the natural frequencies of transverse oscillations was studied. The study was performed for saw disks in the case of oscillations with one, two, and three nodal diameters. It was established that the rigidity of the internal contour of the saw blade has the greatest influence on the natural frequency of transverse oscillations with one nodal diameter

Keywords: circular saw, transverse oscillations, ring plate, natural frequency, elastic fastening

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DETERMINING THE FREQUENCY OF TRANSVERSE OSCILLATIONS OF AN ELASTICALLY FIXED DISK OF A CIRCULAR SAW

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1. Introduction

One of the techniques for splitting wood is its sawing on circular saw machines. The circular saw on such a machine is a multi-blade tool that rotates at a sufficiently high angular speed. The imbalance of the circular saw and the rotary links of the cutting mechanism of the machine, oscillations of the saw shaft [1] cause transverse oscillations of the circular saw disk. Due to the oscillations of the saw blade, the width of the cut increases and the accuracy of wood cutting decreases. Given this, the waste of wood in sawing increases.

In order to increase the accuracy of wood cutting, researchers in works [2, 3] propose to improve the design of the circular saw machine and the toothed crown of the circular saw. However, the accuracy of cutting wood with a machine tool and the durability of a circular saw as a tool are significantly affected by its dynamic properties. The natural frequency of transverse oscillations belongs to the dynamic properties of a circular saw. If the natural frequency of the transverse oscillations of the saw blade coincides with the frequency of its rotation, resonance may occur, and the saw may lose its operational state. Therefore, it is advisable to perform a study of the transverse oscillations of the saw blade since the frequency of its free transverse oscillations is an important characteristic for evaluating dynamic properties.

2. Literature review and problem statement

Transverse oscillations of the saw blade play an important role in the quality of processing during wood cutting and affect its operation [4]. In the cited paper, the transverse vibration of a circular saw blade under constant rotation is investigated by simulation using ANSYS software. The amplitudes of transverse oscillations of the saw blade were determined but the natural frequencies were not defined. To reduce the amplitudes of transverse oscillations of a circular saw, piezoelectric shunt damping of oscillations with an autonomous power source is proposed in [5]. The energy required for this technique of damping is generated from the rotation of the saw. The authors note that as a result of such damping, the vibration of the saw blade decreases and the noise level decreases.

According to [6], transverse oscillations of circular saw blades are an undesirable situation that negatively affects both tool life and wood cutting accuracy. In the cited work, natural frequencies of transverse oscillations were determined for two circular saws using the finite element method, and critical frequencies of their rotation were determined experimentally during cutting. However, the work does not take into account the real conditions of fixing the saw blade.

It is stated in [7] that specific conditions are very often established in mechanical systems with rotating thin disk-shaped tools. These conditions cause unwanted tool lateral deflections and vibration. The vibration excitation mode of disk-shaped instruments is mechanical and depends on rotation. External loads on the tool change over time. In some cases, even self-excitation of disk tool oscillations may occur. The problem can be eliminated by a more rigid design of the disk tool, increasing its thickness. However, an increase in thickness due to numerous technological requirements is unacceptable. Therefore, an electromagnetic lateral damping system is proposed to reduce the vibration of circular saws. This electromagnetic system consists of two electromagnets installed transversely to the housing that carries the ferromagnetic saw blade. The authors claim that the developed system is an effective way to reduce the lateral vibrations of rotating disk tools without the specified values of the frequencies of such vibrations.

The frequency of the natural transverse oscillations of the circular saw disk is influenced by its rolling, which creates additional stresses. According to work [8], the rolling of the saw disk will make it possible to create circular saws with predetermined dynamic properties. In work [8], the frequencies of self-oscillations of saws that were rolled with different pressing forces of the rollers were determined using the finite element method. The results are in good agreement with the experimentally established natural frequencies of oscillations of these saws. However, fixing the internal contour of the saw blade during experimental research did not correspond to the real conditions of mounting a circular saw on the shaft using clamping flanges. The use of rolling circular saw disks to change the natural frequencies of oscillations can also have a negative effect on the dynamic properties of the saws: the creation of excessive tension deforms the saw blade and changes the natural frequencies.

The dependence of the dynamic properties of the circular saw blade on the number of radial slits and temperature distribution was evaluated in [9]. The critical rotational speed of a 300 mm diameter saw is reduced by the presence of more than six 30 mm long radial slots. In the cited work, the natural frequencies of oscillations of circular saws are determined by the finite element method. The influence of the lengths of the radial slits on the natural frequency and deformation of four types of circular saws with an outer contour diameter of 780 mm and a thickness of 5.5 mm was investigated in [10] also using the finite element method. In the cited work, the authors note that the natural frequency of transverse oscillations of a circular saw was sensitive to the length of the slot on its disk. The first natural frequency of oscillations for all four types of saws was less than 50 Hz, that is, it did not depend on the size of the slits and toothed crown. The lengths of the slits had an effect on the natural frequency of the fifth and higher orders. The natural frequency increased with an increase in the length of the slit. The authors calculated the natural frequencies of circular saws at their rotation speed of 1600 rpm. However, the dependence of the natural frequency of circular saws on the speed of rotation has not been investigated.

In [11], the influence of the rotational speed of the circular saw blade, the shape of the slits, and the number of slits on its natural frequencies was investigated and critical rotational speeds were predicted. The fourth-order differential equation obtained on the basis of Hamilton's principle and the Kirchhoff-Lowe hypothesis for thin plates was used for the study, which describes the transverse oscillations of a fixed ring plate rigidly fixed along the inner contour. Next, the results obtained from the analytical solution of the differential equation are compared with the results obtained using the finite element method. Then the finite element method is used to study the calculation model of the circular saw blade. In the cited work, a circular saw is considered as a unit consisting of two parts: a disk and teeth. To analyze the vibration of the saw, the calculation model uses an annular plate, which is free on the outer radius and fixed on the inner radius. The boundary conditions for solving the differential equation correspond to the free outer contour of the plate and the hinged inner contour. The value of the first natural frequencies of transverse oscillations for different number of nodal diameters and nodal circles was obtained for a 2.2 mm thick circular saw disk with contour radii: outer – 150 mm, inner – 12.75 mm. However, in the work, the authors do not take into account the effect on the natural frequency of oscillations of the stiffness of the internal contour of the saw blade.

Determination of the critical speed of rotation of circular saws based on the natural frequencies of transverse oscillations of ring plates of similar sizes was performed in [12]. The authors noted that with an increase in the speed of rotation beyond the permissible one, the quality of the treated surfaces decreases due to increased vibration. The cited paper calculates the critical speeds of rotation of standard circular saws with a flat surface without radial gaps using the finite element method and the classical theory of thin plates. Forms of transverse oscillations and natural frequencies are determined using Bessel functions and polynomial functions. However, the influence of the speed of rotation of the circular saw and the rigidity of fixing the internal contour of the saw on the natural frequencies of its transverse oscillations was not considered in the work.

The calculated models of circular saws in works [11, 12] are circular plates with a free outer contour and a correspondingly fixed inner one. The study of free transverse vibrations of an annular plate that does not rotate was performed in [13]. The round ring plate along the inner contour is hinged or rigidly clamped. The outer contour is pinched or free. In the cited work, frequency equations for ring plates with corresponding methods of fixing their contours were obtained. Depending on the ratio between the radii of the inner and outer contours of the annular plates, the frequency parameters and radii of the nodal rings were obtained in the case of symmetric and asymmetric transverse oscillations. Solving the fourth-order differential equation for transverse vibrations of an annular plate is performed by the method of separation of variables using Bessel functions of various kinds of zero and first order. The frequency parameters for the ring plate depend on the ratio of the radii of its inner and outer contours. For circular saw blades, the ratio between contour radii is usually much smaller than the minimum value specified in [13]. Therefore, it is practically impossible to use the results of the cited work to determine the natural frequencies of oscillations of circular saw blades.

In [14], an algorithm was built for the analytical solution of the problem of self-oscillations of a composite two-stage annular plate with stages of variable (concave) and constant thickness. The authors obtained ratios that allow studying the distribution of deflections and determining the values of the amplitudes of bending vibrations of the annular plate. The forms of oscillations are built on the basis of the provisions of the symmetry and factorization methods devised and developed by the authors of [14]. In the cited work, the frequency equation for a two-stage annular plate of variable thickness, which is rigidly fixed on the inner contour and has a free outer contour, is constructed. The frequency equation is represented in the form of an eighth-order determinant. The authors also used the symmetry method and the factorization method for the general analytical solution of the fourth-order differential equation with variable coefficients, which describes the free axisymmetric oscillations of an annular plate of variable thickness in [15]. The authors considered an annular plate with a rigid fixation of the inner contour and a free outer contour. The solution to the problem of free transverse oscillations of an annular plate is given in Bessel functions of zero and first order from real and imaginary arguments. The first three frequency numbers and forms of transverse oscillations are determined. It is shown that the first three frequency numbers and forms of oscillations decrease with increasing concavity. Usually saw disks are considered as annular plates of constant thickness with certain conditions for fixing contours. Therefore, it is not advisable to use the frequency equations constructed in [14, 15] to study the natural frequencies of saw disks.

In [16], free oscillations of a circular saw disk are considered, taking into account the real conditions of fixing its internal contour on the saw shaft of the machine. In the cited work, a graphical dependence of the first natural frequency for a circular saw of the same standard size was obtained for only one value of the stiffness coefficient of the internal contour fixation on the angular speed of its rotation. However, in [16], the effect of changing the fixing stiffness coefficient on the first and higher frequencies of natural oscillations of other standard sizes of circular saws was not clarified.

The review of the above publications showed that in the study of the vibration frequencies of circular saws and ring plates, the problem of the influence of the elasticity of the internal circuit of the saw blade on the frequency of oscillations remains unsolved. Therefore, it is advisable to conduct a study of transverse oscillations of circular saw disks, taking into account their rotation and the actual conditions of fixing the internal contour on the shaft of the machine.

3. The aim and objectives of the study

The purpose of this work is to determine the frequency of free transverse oscillations of a circular saw blade, taking into account the rigidity of its fastening on the saw shaft and the angular speed of rotation. This will make it possible to predict the critical rotation frequencies of the saw shaft to avoid resonance of the circular saw disk.

To achieve the goal, the following tasks are set:

– to construct the solution to the nonlinear differential equation of the transverse vibrations of the stretched disk of a circular saw under the appropriate boundary conditions;

– to investigate the influence of the rigidity of the fastening of the internal contour of the circular saw disk and the angular speed of its rotation on the change in the natural frequency of the transverse oscillations of the disk.

4. The study materials and methods

The object of our study is a circular saw blade.

The hypothesis of the study assumes a significant influence exerted on the frequency of transverse oscillations of the circular saw disk by the actual conditions of fixing the internal contour.

Given the small height of the teeth of a circular saw compared to the outer radius of the saw disk, the influence of the toothed crown on the natural frequency of transverse oscillations was neglected. An annular plate of constant thickness was used as the calculation model of the circular saw blade.

When the circular saw machine was idling without cutting wood, the outer contour of the saw blade was considered free. The inner contour of the saw disk was clamped with flanges. Rubber gaskets were placed between the flanges and the disk of the circular saw. In any case, the inner contour of the saw blade would have some flexibility in the direction perpendicular to the plane of the blade. Therefore, the inner contour of the circular saw disk was considered elastically fixed. In accordance with the accepted assumptions, the boundary conditions of the boundary value problem on the transverse oscillations of the annular plate were formulated further.

To determine the natural frequencies of the saw blade, the theory of transverse vibrations of the plates was applied. The theory is based on the Kirchhoff-Levy hypotheses, taking into account the forces in the middle surface of the plate. As a result of the rotation of the saw blade with a high angular speed due to the action of centrifugal forces, normal radial and tangential stresses arise in it. Radial normal stresses are directed from the center of the plate at stretching. These stresses can be accounted for through the radial force in the median plane of the stretched annular plate.

The material for research was steel with Poisson's ratio μ=0.29 and Young's modulus E=1.93·10^{11} Pa. Density of steel ρ=8000 kg/m^3.

The Bubnov-Galyorkin numerical method for solving the boundary-value problem was used to construct the solution to the fourth-order nonlinear differential equation of transverse vibrations of an annular plate stretched by centrifugal forces. The first step in the implementation of this method is the selection of basis functions that satisfy the boundary conditions of the free outer contour of the annular plate and the elastically supported inner contour. The solution algorithm according to the selected method was implemented in the Maple 15 mathematical environment (Canada).

5. Results of the study of transverse vibrations of a circular saw disk

5. 1. Construction of the solution to the differential equation of transverse oscillations of a stretched disk of a circular saw

The calculation scheme of a circular saw for the study of transverse vibrations is adopted in the form of an annular plate (Fig. 1). The outer contour of the plate with radius R_1 is free, and the inner contour with radius R_2 was considered elastically fixed with a linear stiffness coefficient c. Such a calculation scheme of the saw blade corresponds to the actual fastening of the internal contour of the machine on the saw shaft with a face plate with rubber gaskets [16].

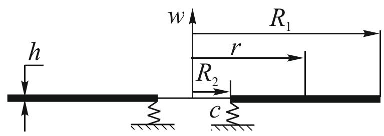

Fig. 1. Round saw design diagram

For a stretched annular plate, the nonlinear differential equation of transverse oscillations in polar coordinates r, θ takes the following form [17]:

$$D\Delta\Delta w + N(r)\Delta w + m\frac{\partial^2 w}{\partial t^2} = 0, \tag{1}$$

where $w=w(r, \theta, t)$ is the deflection of the plate;

$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$ is the Laplace operator in polar coordinates r, θ;

$D = \frac{Eh^3}{12(1-\mu^2)}$ is the cylindrical stiffness of the plate;

E is Young's modulus and μ is the Poisson ratio of the plate material;

h is the plate thickness;

$m=\rho h$ is the mass per unit area of the plate;

ρ is the density of the material;

$N(r)$ is the radial force in the middle surface of the plate.

The force $N(r)$ depends on the angular speed of rotation of the plate. This force is given as follows:

$$N(r) = \sigma(r)\cdot h, \tag{2}$$

where $\sigma(r) = \frac{3+\mu}{8}\cdot\frac{\rho\omega^2}{g}\left(R_1^2 + R_2^2 - \frac{R_1^2 R_2^2}{r^2} - r^2\right)$ is the radial stress in the disk of a circular saw due to the action of centrifugal forces, the expression for which is obtained from the well-known problem of the theory of elasticity about the stress in an annular plate rotating with an angular velocity ω, R_1 is the outer radius of the saw disk (without a gear ring), R_2 is the inner the radius of the saw disk, which is equal to the outer radius of the clamping flanges, g is the acceleration of the force of gravity.

The boundary conditions for solving equation (1) are written taking into account the technique of fastening the circular saw in the cutting mechanism of the circular saw machine:

– at $r=R_1$, the combined transverse force Q_r and the bending moment M_r are equal to zero:

$$M_r = -D\left[\frac{\partial^2 w}{\partial r^2} + \mu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right] = 0;$$

$$Q_r = \frac{\partial^3 w}{\partial r^3} + \frac{1}{r}\frac{\partial^2 w}{\partial r^2} - \frac{1}{r^2}\frac{\partial w}{\partial r} - \frac{2-\mu}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{\mu}{r^2}\frac{\partial^3 w}{\partial r \partial \theta^2} = 0; \tag{3}$$

– at $r=R_2$:

$$Q_r + cw = 0; \quad M_r = 0, \tag{4}$$

where c is the coefficient of linear stiffness of the internal contour of the saw blade.

Equation (1) was solved using the Bubnov-Galyorkin numerical method. The deflection of the saw blade is given as follows:

$$w(r,\theta,t) = W(r,\theta)a_0\cos(\omega_0 t - \varphi_0), \tag{5}$$

where $W(r, \theta)$ is a function of the natural forms of saw disk oscillations, a_0 is the amplitude of saw disk oscillations, ω_0 is the natural frequency of oscillations, and φ_0 is the initial phase of oscillations.

After taking into account (5) in (1), the differential equation for the natural forms of oscillations is obtained:

$$D\Delta\Delta W(r,\theta) + N(r)\Delta W(r,\theta) - m\omega_0^2 W(r,\theta) = 0. \tag{6}$$

The eigenforms function of oscillations is represented as follows:

$$W(r,\theta) = aw_1 + bw_2, \tag{7}$$

where a, b are unknown constants (variation parameters). The functions w_1, w_2 are given in the following form:

$$w_1 = \left(1 + A_1 r + A_2 r^2 + A_3 r^3 + A_4 r^4\right)\sin\lambda\theta;$$

$$w_2 = \left(1 + B_1 r^2 + B_2 r^3 + B_3 r^4 + B_4 r^5\right)\sin\lambda\theta, \tag{8}$$

where λ is the number of nodal diameters, A_1, ..., A_4, B_1, ..., B_4 – coefficients. These coefficients are chosen so that the boundary conditions (3), (4) are fulfilled. After taking into account functions (8) in the expression for $W(r, \theta)$, further substitution in equation (6) and integration, we obtained:

$$\int_{R_2}^{R_1}\int_0^{2\pi}\left[\begin{array}{l} D\Delta\Delta W(r,\theta) + \\ +N(r)\Delta W(r,\theta) - \\ -m\omega_0^2 W(r,\theta) \end{array}\right]\delta W\cdot r\,dr\,d\theta = 0,$$

where δW is the variation of the function of the eigenforms of the saw disk oscillations.

Variations δa and δb are independent. Then the factors near the variational parameters a and b are equal to zero. Taking this into account, a system of linear equations with respect to a and b is obtained:

$$\begin{cases} K_1 a + K_2 b = 0; \\ K_3 a + K_4 b = 0, \end{cases} \tag{9}$$

where

$$K_1 = \int_{R_2}^{R_1}\int_0^{2\pi}\left[\begin{array}{l} D\Delta\Delta w_1(r,\theta) + N(r)\Delta w_1(r,\theta) - \\ -m\omega_0^2 w_1(r,\theta) \end{array}\right] w_1 r\,dr\,d\theta;$$

$$K_2 = \int_{R_2}^{R_1}\int_0^{2\pi}\left[\begin{array}{l} D\Delta\Delta w_2(r,\theta) + N(r)\Delta w_2(r,\theta) - \\ -m\omega_0^2 w_2(r,\theta) \end{array}\right] w_1 r\,dr\,d\theta;$$

$$K_3 = \int_{R_2}^{R_1}\int_0^{2\pi}\left[\begin{array}{l} D\Delta\Delta w_1(r,\theta) + N(r)\Delta w_1(r,\theta) - \\ -m\omega_0^2 w_1(r,\theta) \end{array}\right] w_2 r\,dr\,d\theta;$$

$$K_4=\int_{R_2}^{R_1}\int_0^{2\pi}\left[\begin{matrix}D\Delta\Delta w_2(r,\theta)+N(r)\Delta w_2(r,\theta)-\\-m\omega_0^2 w_2(r,\theta)\end{matrix}\right]w_2 r\,\mathrm{d}r\mathrm{d}\theta.$$

From the equality of the determinant of system (9) to zero, the natural cyclic frequency ω_0 of the transverse oscillations of the circular saw is determined:

$$\begin{vmatrix}K_1 & K_2\\ K_3 & K_4\end{vmatrix}=0. \qquad (10)$$

The described algorithm for determining the frequency of transverse oscillations of a circular saw disk was developed in the Maple 15 mathematical environment. The frequency equation obtained in this environment takes the following form:

$$C_1\omega_0^2+ +\left(C_2\omega^2-C_3\right)\omega_0- -C_4\omega^2+C_5\omega^4+ +C_6=0, \qquad (11)$$

where $C_i,\ i=\overline{1,6}$ are constants, the values of which are determined by the geometric dimensions of the saw blade and the physical and mechanical characteristics of its material.

5. 2. Studying the influence of the rigidity of the attachment and rotation of the saw blade on natural frequencies

The solutions to the frequency equation (11) are the values of the natural frequencies ω_0 of the transverse oscillations of the disks of circular saws at different values of c – the stiffness coefficient of the fastening of the internal contour of the saw disk. In the absence of rotation of the saw blade, the values of cyclic ω_0 and natural f frequencies for three saw blades with different radii of the outer contour and the radius of the inner contour R_2 of 12.7 mm are given in Tables 1–3.

An increase in the thickness of saw disks with the indicated radii of the outer contours leads to an increase in the frequency of their natural oscillations.

Plots of the dependence of the first natural frequency ω_0 of the transverse oscillations of circular saw blades 2.2 mm thick at $c=10^8$ N/m on the angular speed ω of the saw blade rotation are shown in Fig. 2.

With an increase in the angular speed of rotation of saw disks from zero to 320 rad/s, the natural cyclic frequencies of their transverse oscillations with one nodal diameter increase nonlinearly.

Table 1

Values of natural circular frequencies ω_0 and frequencies f of circular saw blades with one nodal diameter $\lambda=1$

Coefficient c, N/m	Thickness h, mm	Outer contour radius R_1					
		150 mm		200 mm		250 mm	
		ω_0, rad/s	f, Hz	ω_0, rad/s	f, Hz	ω_0, rad/s	f, Hz
10^6	2.2	395.98	63.02	217.74	34.65	136.93	21.79
	2.4	431.98	68.75	237.53	37.8	149.38	23.77
	2.6	467.98	74.48	257.33	40.96	161.83	25.76
	2.8	503.98	80.21	277.12	44.11	174.28	27.74
	3.0	539.97	85.94	296.92	47.26	186.73	29.72
10^7	2.2	473.09	75.29	272.65	43.39	183.31	29.17
	2.4	516.1	82.14	297.44	47.34	199.97	31.83
	2.6	559.11	88.99	322.22	51.28	216.63	34.48
	2.8	602.11	95.83	347.01	55.23	232.30	36.97
	3.0	645.12	102.70	371.80	59.17	249.96	39.78
10^8	2.2	475.58	75.69	276.21	43.96	187.64	29.86
	2.4	518.81	82.57	301.32	47.96	204.70	32.58
	2.6	562.04	89.45	326.43	51.95	221.76	35.29
	2.8	605.28	96.33	351.54	55.95	238.82	38.01
	3.0	648.51	103.20	376.65	59.95	255.88	40.72
10^9	2.2	475.72	75.71	276.53	44.01	188.07	29.93
	2.4	518.97	82.60	301.67	48.01	205.17	32.65
	2.6	562.22	89.48	326.8	52.01	222.27	35.38
	2.8	605.47	96.36	351.94	56.01	239.37	38.10
	3.0	648.71	103.20	377.08	60.01	256.46	40.82

Table 2

Values of natural circular frequencies ω_0 and frequencies f of circular saw blades with two nodal diameters $\lambda=2$

Coefficient c, N/m	Thickness h, mm	Outer contour radius R_1					
		150 mm		200 mm		250 mm	
		ω_0, rad/s	f, Hz	ω_0, rad/s	f, Hz	ω_0, rad/s	f, Hz
10^6	2.2	826.08	131.47	450.94	71.77	279.03	44.41
	2.4	901.18	143.43	491.93	78.29	304.4	48.45
	2.6	976.28	155.38	532.92	84.82	329.77	52.48
	2.8	1051.37	167.33	573.92	91.34	355.13	56.52
	3.0	1126.47	179.28	614.91	97.87	380.50	60.56
10^7	2.2	821.81	130.8	445.66	70.93	274.53	43.69
	2.4	896.52	142.69	486.17	77.38	299.49	47.67
	2.6	971.23	154.58	526.68	83.82	324.44	51.64
	2.8	1045.94	166.47	567.2	90.27	349.40	55.61
	3.0	1120.65	178.36	607.71	96.72	374.36	59.58
10^8	2.2	820.42	130.57	444.76	70.79	274.25	43.65
	2.4	895.00	142.44	485.19	77.22	299.18	47.62
	2.6	969.59	154.32	525.62	83.66	324.12	51.59
	2.8	1044.17	166.18	566.06	90.09	349.05	55.55
	3.0	1118.75	178.05	606.49	96.53	373.98	59.52

Table 3

Values of natural circular frequencies ω_0 and frequencies f of circular saw blades with three nodal diameters λ=3

Coefficient c, N/m	Thickness h, mm	Outer contour radius R_1					
		150 mm		200 mm		250 mm	
		ω_0, rad/s	f, Hz	ω_0, rad/s	f, Hz	ω_0, rad/s	f, Hz
10^6	2.2	1596.29	254.06	876.90	139.56	546.5	86.98
	2.4	1741.41	277.15	956.62	152.25	596.18	94.88
	2.6	1886.52	300.25	1036.34	164.94	645.86	102.79
	2.8	2031.64	323.35	1116.06	177.63	695.54	110.7
	3.0	2176.76	346.44	1195.77	190.31	745.22	118.61
10^7	2.2	1593.54	253.62	876.27	139.46	546.62	86.00
	2.4	1738.41	276.68	955.93	152.14	596.31	94.91
	2.6	1883.28	299.73	1035.59	164.82	646.00	102.81
	2.8	2028.15	322.79	1115.25	177.5	695.69	110.72
	3.0	2173.02	345.85	1194.91	190.18	745.39	118.63
10^8	2.2	1591.14	253.24	875.10	139.28	546.11	86.92
	2.4	1735.78	276.26	954.66	151.94	595.76	94.82
	2.6	1880.43	299.28	1034.21	164.60	645.4	102.72
	2.8	2025.08	322.3	1113.77	177.26	695.05	110.62
	3.0	2169.73	345.32	1193.32	189.92	744.7	118.52

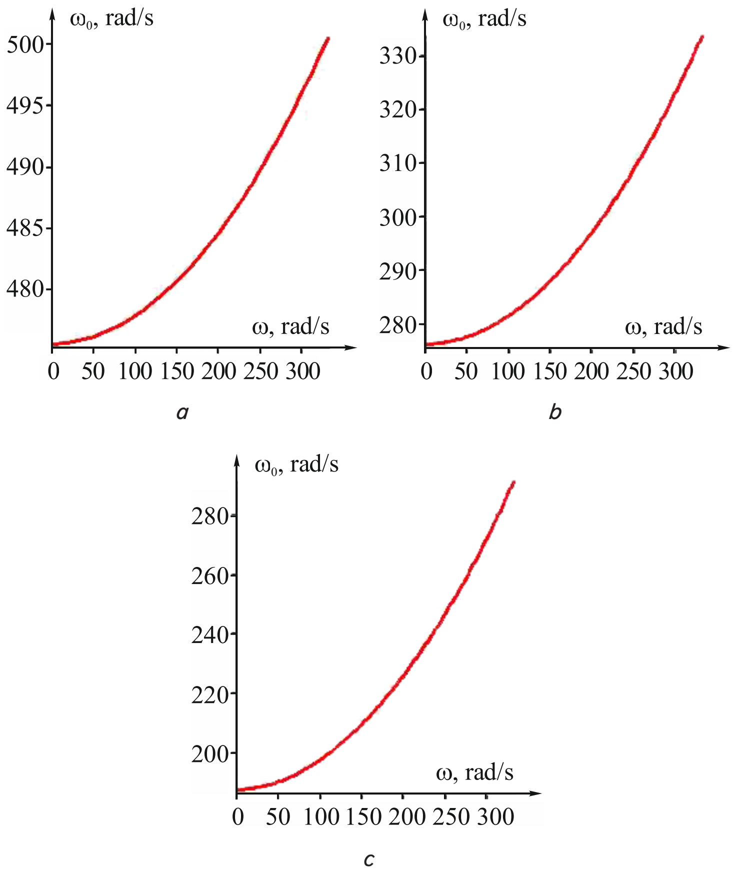

Fig. 2. Plots of changes in the first natural frequency of transverse oscillations of circular saw blades at c=10^8 N/m depending on the angular speed of rotation: a – R_1=150 mm, b – R_1=200 mm; c – R_1=250 mm

6. Discussion of results of determining the natural frequencies of transverse oscillations of saw disks

The numerical solution to the differential equation of the transverse vibrations of the disk stretched by the rotation of a circular saw has been constructed. The adopted boundary conditions take into account the elastic fastening of the inner contour using the stiffness coefficient. This has made it possible to investigate the influence of two factors on the natural frequencies of the saw blade: the rigidity of the internal circuit fastening and the angular speed of rotation of the saw shaft. The natural frequencies of circular saw disks are determined from the frequency equation (11) for three values of the outer radius of the saw disk and five values of its thickness.

In contrast to [9, 11], where the natural frequencies of circular saws are determined under ideal boundary conditions, the constructed numerical solution to the differential equation of the transverse vibrations of a stretched disk of a circular saw makes it possible to take into account the actual conditions of its fastening.

The dynamic model of transverse vibrations of the circular saw disk is implemented as a program in the Maple 15 mathematical environment and can be used by scientific and technical workers engaged in the design and operation of circular saw machines to predict the critical speeds of rotation of saw shafts.

It was established that the increase in the natural oscillation frequencies for all values of the stiffness coefficient of the internal contour fastening occurs with an increase in the thickness of the saw disks. For saw disks with outer contour radii of 150 mm, 200 mm, 250 mm at λ=1 (Table 1), the natural frequency increases by 26.7 % with an increase in the thickness of the saw disks from 2.2 mm to 3 mm.

With an increase in the fastening stiffness coefficient from c=10^6 N/m to c=10^9 N/m (Table 1), the frequency of transverse oscillations of saw disks does not increase uniformly. For a thickness of 2.2 mm, the circular frequency of a disk with an outer radius of R_1=150 mm increased by 16.76 %, when R_1=200 mm – by 21 %, when R_1=250 mm – by 27.2 %. Increasing the radius of the outer contour of the saw blade from 150 mm to 250 mm for all thickness values reduces the frequency of oscillations by 65.4 %.

An increase in the stiffness coefficient of the internal contour fastening from c=10^8 N/m to c=10^9 N/m practically does not affect the change in the frequency of transverse oscillations. The difference between the values of the frequencies for the specified values of the stiffness coefficient is 0.03 % (Table 1). It is assumed that the value c=10^8 N/m corresponds to the rigid fixation of the inner contour of the saw blade. A further increase in the value of the coefficient does not lead to an increase in frequency. Therefore, the calculation of frequencies at λ=2 and λ=3 was performed for values of c=10^6–10^8 N/m (Tables 2, 3). For transverse oscillations with two nodal diameters (Table 2, λ=2), with an increase in the stiffness coefficient, the natural frequencies of saw disks decrease by 0.68 % for all disk thickness values.

For oscillations with two nodal diameters (Table 2, λ=2), an increase in the thickness of the disks for all values of the stiffness coefficient of the internal contour fastening also leads to an increase in the frequencies of natural oscillations by 26.7 %, as in the case of oscillations with one nodal diameter. In the case of oscillations with three nodal diameters (Table 3, λ=3), with an increase in the rigidity of the fastening, the natural frequencies of the saw disks decrease by 0.32 %. Therefore, the stiffness of the internal contour of the saw blade has the greatest influence on the natural frequency of oscillations in the case of oscillations at the lowest frequency with one nodal diameter.

The reliability of our values of the circular and natural frequencies of circular saw blades is confirmed by the fact that for a circular saw with contour radii: external – 150 mm, internal – 12.7 mm, thickness 2.2 mm for the value of the stiffness coefficient $c=10^7$ N/m. The calculated frequencies for one nodal diameter (λ=1, Table 1) are equal to ω_0=473.09 rad/s, f=75.29 Hz, and in work [11], under the condition of rigid fastening of the internal circuit, the following frequency values are given – ω_0=475.87 rad/s, f=75.73 Hz. The difference between the results is 0.53 %. For two nodal diameters (λ=2, Table 2), the calculated frequencies are ω_0=821.81 rad/s, f=130.8 Hz, in work [11] – ω_0=808.12 rad/s, f=128 ,62 Hz. The difference between the results is 1.67 %. For three nodal diameters (λ=3, Table 3), the calculated frequencies are ω_0=1593.54 rad/s, f=253.62 Hz, in work [11] – ω_0=1811.9 rad/s, f=288,38 Hz. The difference between the results is 12 %. Typically, the rotation frequency of circular saws in wood sawing machines is 500–3000 rpm, which corresponds to an angular velocity of 52.3–314 rad/s. The frequencies of natural oscillations of the investigated saw disks at λ=3 acquire values greater than 314 rad/s and are in the resonance zone. Therefore, the difference of 12 % between the results is not fundamental. Therefore, at the two lower frequencies, the obtained results coincide with an accuracy of less than two percent with the known results of other researchers.

As the angular speed of rotation of the saw shaft increases from 0 to 314 rad/s, the circular frequency of the saw disks increases. Accordingly, for the radii of the outer contours of the disks, the increase is: R_1=150 mm – 4.88 % (Fig. 2, *a*); R_1=200 mm (Fig. 2, *b*) – 16.3 %; R_1=250 mm (Fig. 2, *c*) – 33.93 %.

Using the proposed approach to the study of transverse oscillations of saw blades, it is possible to determine the natural oscillation frequencies of circular saw blades of various diameters and thicknesses, in particular, large-diameter circular saw blades. However, in recent years, log sawing circular saw machines have been replaced by band saw machines for sawing logs in the woodworking industry. Therefore, in this work, the study of natural transverse vibrations of saws of large diameters is not considered.

Neglecting the effect of the gear crown of a circular saw on the natural frequencies of oscillations is a certain shortcoming of the study, which does not significantly affect the results.

Therefore, the study of transverse oscillations of saw disks as annular plates, taking into account the actual technique of fixing the internal contour and their rotation, allows us to investigate the influence of these factors on the natural frequency of transverse oscillations. Real techniques of fastening the inner contour of the saw blade on the shaft of the machine using clamping flanges with rubber gaskets require further experimental research. Such a study could make it possible to determine the actual values of the stiffness factor of fastening the internal contour of the disk.

7. Conclusions

1. The solution to the nonlinear differential equation of the transverse vibrations of the stretched disk of a circular saw was constructed using a numerical method under the following boundary conditions: the outer contour of the saw disk is free; the inner contour is elastically fixed. The frequency equation was obtained and the natural frequencies of transverse oscillations of circular saw blades of different thickness with the same radius of the inner contour and three values of the radii of the outer contour were determined.

2. The impact of the rigidity of the internal contour of the circular saw disk and the angular speed of its rotation on the change in the natural frequency of transverse oscillations was analyzed. It was established that with an increase in the rigidity of the internal contour fixation by 1000 N/m, the natural frequencies of oscillations of saw disks for one nodal diameter increase to 27.2 %. For transverse oscillations with two or three nodal diameters, changes in the natural frequencies of disk oscillations at the considered in the work stiffness values are insignificant (up to 1 % decrease in frequency). For oscillations with one nodal diameter, the angular speed of rotation of the saw blade increases its own frequency of transverse oscillations to 33.93 %.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

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