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SYNTHESIS OF TWO-MASS ELECTROMECHANICAL SYSTEMS WITH CASCADE CONNECTION OF FRACTIONAL-ORDER CONTROLLERS

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The research focused on two-mass electromechanical systems widely utilized in industry. The challenge addressed in this work was to improve the synthesis of controllers for such systems to simplify it and enhance the quality of transition processes. Traditionally, the synthesis of control system loops for these systems was carried out using integer controllers and standard forms. However, this approach led to the synthesis of complex integer controllers that are difficult to implement. To overcome this issue, an original approach to the synthesis of control system loops based on the fractional characteristic polynomial is proposed. The fractional characteristic polynomial ensures the desired quality of the transition process given the implementation of a specified structure of the fractional controller. A new method of structural-parametric synthesis of fractional-order controllers is developed for the case of their cascade connection in multi-loop two-mass electromechanical systems. Additionally, an algorithm for synthesizing fractional-order controllers for the corresponding control loops is presented. This enabled the structural-parametric synthesis of fractional-order controllers for a two-mass electromechanical system with the cascade connection of controllers. Such an approach provides better quality of transition processes compared to classical integer controllers, simplifies the synthesis, and thereby enhances the quality of the synthesized systems. The impact of the synthesized fractional-order controllers using the proposed approach on the dynamic properties of the two-mass «thyristor converter – motor» system was investigated. The research results demonstrated the practical applicability of fractional controllers designed using the proposed method for the synthesis of automatic control systems of two-mass electromechanical systems in the industry

Keywords: two-mass electromechanical system, fractional-order controllers, fire lift

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1. Introduction

Electromechanical systems (EMSs) must meet the requirements imposed on them by the executive or production mechanism or technological process. Various methods of EMS synthesis are used for this purpose. A mandatory requirement for any synthesis method is to take into account the dynamic properties of the control object, because it usually imposes certain requirements and significantly affects the dynamic and static characteristics of this EMS.

In practice, one often has to deal with structural-parametric methods of EMS synthesis. Structural-parametric synthesis, given the output objective function, input control action, and structure of the main part of the system (object), determines the number, type, and place of input of control connections. In addition, this synthesis option determines the general structure of control devices, characteristics and numerical values of parameters. A feature of such systems is the use of typical controllers. The main task for synthesizing EMS comes down to selecting a typical controller that provides a given control law and finding its parameters.

The optimal systems of subordinate regulation (SSR) proposed by Kesler are currently dominant. In such systems, the principle of sequential correction is implemented for each coordinate [1, 2]. Traditionally, SSRs are synthesized based on two standard forms of root distribution of the characteristic equation, namely: according to the modular (technical) optimum and according to the symmetric optimum. At the same time, the principle of doubling the time constant of each subsequent external loop is the basis for the synthesis and construction of the SSR. The task of each controller in such an SSR is to ensure the desired dynamic characteristics of its control coordinate. If we change the optimization criterion for SSR loops in accordance with a certain standard form of the integer order, then we will get the corresponding controllers in each loop [3, 4]. The obtained controllers will provide the necessary dynamic characteristics according to the selected form of root distribution of the characteristic equation on the complex plane.

There are two groups of SSR. One group is systems whose transfer functions have no zeros, and the second group is systems whose transfer functions have both zeros and poles. The SSR synthesized for a symmetric or modular optimum does not take into account the zeros of the transfer function. In practice, a method of eliminating zeros of the electromechanical system transfer function by means of appropriate filters connected to the system input is widely used. It is clear that the whole range of standard forms of transition functions can be obtained, if there is no problem related to the reduction of the speed and gain margin of the system. Thus, in the SSR configured for a symmetrical optimum, the introduction of a first-order filter at the system input leads to a decrease in the gain margin. The presence of a second- and higher-order filter often makes it problematic to use synthesized systems.

An alternative option for constructing an SSR is to use the generalized characteristic polynomial (GCP) synthesis method [5]. This method allows you to adjust the SSR to any standard transition function of the output control coordinate of a given speed, without complicating it, without reducing the speed and gain margin of the system. However, it is designed for a generalized characteristic polynomial of integer order.

As research shows, the use of fractional-order controllers in EMS also leads to improved quality of transition processes. This makes it possible to increase the level of stability compared to similar systems that use conventional (integer-order) controllers. The implementation of fractional-order controllers for the management of production processes, instead of conventional integer-order controllers, can be more widely used in the case of solving scientific and applied problems related to the synthesis and implementation of such systems, as well as the development of tuning methods. However, the problem of synthesis and implementation of cascaded fractional-order controllers, as well as the study of their capabilities in automated EMS, requires further development and scientific research.

This is especially true for EMS that have elastic connections and are often found in executive and production mechanisms, such as long shaft mechanisms, lifting and turning mechanisms for the fire truck cradle, rolling mills, conveyors, paper and textile machines. For the analysis of such systems, models of two-mass systems are used.

The boom of the lifting mechanisms of truck lifts, including fire truck lifts, is not absolutely rigid. Due to external influences, elastic deformations occur when the cradle is moved. For example, when lifting the cradle to a height of 50–60 m, the amplitude of its oscillation can reach 3 meters. Such an object is considered as a two-mass system. The performance of the automatic control system for the movement of the arc steel melting furnace electrodes affects its electrical and technical-economic indicators. To improve them, the automatic control system for the movement of electrodes must provide the given dynamic characteristics of the output coordinate. The dynamic characteristics of the electromechanical system are also affected by the elastic properties of the kinematic circuit elements of the executive mechanism. Such an object is also considered as a two-mass system.

Therefore, the importance of this work in this context lies in the need to develop methods for synthesizing two-mass systems that can be described by fractional-order transfer functions. Such systems arise in the case when the control object due to its complexity is easier to describe by fractional order in a more contact form, or when the desired standard form corresponds to a characteristic fractional-order polynomial.

Therefore, research on the synthesis of two-mass electromechanical systems with cascade connection of fractional-order controllers, given that such systems are quite common and the technical implementation of such controllers has already been carried out and investigated, is relevant.

2. Literature review and problem statement

In [6], the method of synthesis of the generalized characteristic polynomial for fractional-order EMS is modernized and the algorithm for the synthesis of fractional-order controllers of the corresponding control loops is given. The proposed approach to the synthesis of SSR loops based on the fractional characteristic polynomial ensures the desired quality of the transition process, provided that a certain structure of the fractional controller is implemented, which depends on the transfer function of the control object. Fractional-order controllers provide better quality of transition processes compared to integer-order controllers and thereby increase the efficiency of synthesized systems. The SSR design has a significant advantage over other systems, due to the ease of setting up each of the loops, as well as the possibility of implementing the restriction of the control coordinates. As an example, a two-loop «thyristor converter – motor» SSR with current and speed loops was synthesized, and the effect of synthesized fractional-order controllers on the dynamic properties of this SSR was investigated. The studies showed the possibility of implementing SSR controllers, which combine loops with integer- and fractional-order transfer functions, as well as systems with only fractional-order loops. However, these studies concerned only a single-mass motor speed SSR, without taking into account the two-mass nature of the object, which can often occur in practice.

The paper [7] considers the transition functions of the motor and the cradle angular speed controllers, which were obtained in the process of synthesis by the method of generalized characteristic polynomial of a two-mass three-loop SSR by turning the cradle of the fire truck. They provided an aperiodic transition process of cradle turning and low sensitivity, in an established mode, to disturbances. However, the controllers have a high order and turned out to be quite complex in practical implementation. Therefore, it is proposed to replace these controllers by approximation by evolutionary methods with more compact $PI^{\lambda}D^{\mu}$ controllers or fractional-order controllers. The studies on digital models confirm the effectiveness of replacing high-order motor and cradle angular speed controllers with $PI^{\lambda}D^{\mu}$ controllers, the transfer functions of which are determined by approximating the transition functions of the controllers using a genetic algorithm. However, the synthesis of such SSR was carried out by the integer-order GCP method.

In [8], based on the general theory of fractional-order derivatives and integrals, the application of the Caputo-Fabrizio operator to improve the mathematical model of a two-mass long-shaft and lumped parameter system was analyzed. The approach proposed in the paper allows taking into account elastic vibrations of the shaft of an electric drive mechanism. On this basis, it is shown that the use of a fractional-order integrator in the elastic moment model makes it possible to reproduce real transition processes of the system. This mathematical model of the two-mass system provides the required accuracy. However, this work does not say anything about the synthesis of controllers for this system.

The advantages of fractional-order controllers over integer ones are mentioned in a number of works. In [9], an internal fractional-order controller for a time-delayed fractional-order model is proposed to provide the desired stability margin of a DC servo system. In [10], it is proposed to synthesize an optimal fractional-order controller by direct synthesis for a fractional-order model. The time domain characteristics of an optimal closed-loop reference fractional-order transfer function are compared to those of an optimal second-order integer reference transfer function. The fractional-order controller outperforms the integer analog by the integral squared error criterion. However, only one feedback loop is considered in the paper.

In [11], a model of a two-mass fractional-order system was developed. To dampen elastic oscillations in this system, a fractional-order PD^{λ} controller was synthesized. In the work, only parametric synthesis of fractional controller was carried out.

In [12], using the Caputo-Fabrizio operator, the effect of replacing integer derivatives with fractional ones in a linear model of a two-mass system is shown. The change in the parameters of a fractional $PI^{\lambda}D^{\mu}$ controller compared to a classic PID controller is analyzed. The paper demonstrates the effect on the transition characteristics of the system using a fractional-order $PI^{\lambda}D^{\mu}$ controller.

The paper [13] presents the application of a fractional-order $PI^{\lambda}D^{\mu}$ controller and an active interference controller for a nonlinear two-mass system. A complete mathematical model in the state space for a nonlinear two-mass system was developed. A fractional-order $PI^{\lambda}D^{\mu}$ controller and an active interference controller were synthesized. An observer was used to determine the control coordinates when constructing the active interference controller. During the synthesis of the fractional controller, the system was considered as a single-loop system, and feedback was formed according to the speed of the first mass.

In [14], a two-mass drive speed control system is considered. PID and fractional-order $PI^{\lambda}D^{\mu}$ are implemented for this system. Fractional-order $PI^{\lambda}D^{\mu}$ was synthesized by the Taylor series expansion method. However, as in [13], during the synthesis of the fractional controller, the system was considered as a single-loop system, and feedback was formed based on the motor speed.

The conclusion made after the analysis of the latest literary sources shows that the use of fractional calculus is considered one of the most promising strategies in the development and research of modern electromechanical systems. It has been proven that fractional-order controllers provide a higher quality of transition processes compared to integer-order ones. The results of experimental tests of fractional-order controllers developed according to the proposed methods confirmed their effectiveness in expanding the possibilities of regulation compared to traditional PID controllers.

3. The aim and objectives of the study

The aim of the study carried out in this paper is to develop a method of structural-parametric synthesis of fractional-order controllers in multi-loop two-mass electromechanical systems, provided their cascade connection. This opens up opportunities for creating new and modernizing existing two-mass electromechanical systems using cascade connection of controllers, which have an expanded range of dynamic properties and correspond to the desired fractional-order forms.

To achieve the aim, the following objectives were accomplished:

– to modernize the method of synthesis of a generalized characteristic polynomial for two-mass electromechanical systems using the desired fractional-order forms and fractional-order controllers;

– to synthesize armature current and DC motor speed controllers of a two-mass electromechanical system using both the standard integer form and the desired fractional form, and investigate the resulting dynamic characteristics;

– to synthesize cascaded torque and second speed controllers of the mechanism using the desired fractional form, and investigate the dynamic characteristics of the synthesized two-mass electromechanical system.

4. Materials and methods

The object of the study in this paper is the processes of coordinate regulation in a two-mass electromechanical system.

The main hypothesis of the study is that it is possible to take as a basis the method of structural-parametric synthesis of controllers in multi-loop two-mass electromechanical systems, provided their cascade connection. The next step is to modernize it by applying the desired fractional-order forms and fractional controllers, which will expand the range of dynamic properties of the system coordinates. The main assumption will be to study the linear model of a two-mass electromechanical system.

To develop and study a model of a two-mass electromechanical system with controllers obtained by conducting the proposed structural-parametric synthesis, provided their cascade connection, the MATLAB package (USA) was used.

MATLAB is a powerful application package developed by The MathWorks company for numerical analysis in various fields, including electromechanics and energy. MATLAB is unique in having its own programming language, which facilitates complex calculations and algorithm development, and the Simulink simulation environment. In addition, MATLAB provides the ability to create user interfaces that allow you to interact with programs and algorithms in a convenient way.

As far as research in electromechanics is concerned, MATLAB is becoming an indispensable tool for studying and modeling various electrical and mechanical systems and their control systems. Due to its diverse mathematical functions and libraries, MATLAB is used to solve problems in the analysis, synthesis and optimization of electromechanical systems, including the study of electric motors, power converters and other devices. Therefore, this package has become an integral part of modern research in the field of electromechanics.

5. Results of research on a two-mass electromechanical system with cascade connection of integer- and fractional-order controllers

5. 1. Results of modernization of the generalized characteristic polynomial synthesis method for two-mass electromechanical systems

It is known that there are also such EMSs whose control object is described precisely by fractional-order characteristic polynomials q [15, 16]. In this case, the synthesis of such a fractional-order EMS can obviously be carried out similarly to an integer-order SSR. Similarly, the synthesis of fractional-order EMS is carried out in the case when the ACS (automatic control system) provides for the use of a fractional controller.

In this case, obviously the desired optimization criterion will correspond exactly to the desired fractional-order characteristic polynomial $H_d(s)$. For the desired fractional-order characteristic polynomial, the transition functions of the control coordinate are known, and therefore, the selection of $H_d(s)$ is based on the necessary requirements for the dynamic characteristics of the control coordinate under consideration.

The work proposes to improve the known GCP method [5, 6] for selecting both the structure and parameters of fractional-order controllers. The known fractional-order $PI^{\lambda}D^{\mu}$ controller is one option of a wider class of fractional-order controllers. First, it is necessary to select the desired fractional-order form according to the desired transition process. The work [6] proposes the following form, in particular. The expression of its transfer function (TF) is as follows:

$$W_{s1}(s)=\frac{\omega_{o1}/K_1}{s^q+\omega_{o1}}, \qquad (1)$$

where ω_{o1} is the geometric mean root of the desired form, which determines the system speed, q is the fractional order of the characteristic polynomial, K_1 is the feedback gain for the desired control coordinate.

The algorithm of the proposed GCP method for synthesizing a wide range of fractional-order controllers is actually as follows:

1. For a given block diagram of the closed loop under study, it is necessary to determine its TF.

2. By dividing the numerator and denominator of the obtained TF by the numerator, an expression will be obtained that in its structure already resembles a certain desired fractional form.

3. Choose a certain fractional form as desired, for example (1), based on the desired parameters of the transition process: (δ, $t_{0.95}$). Next, the expression found in the previous point of the algorithm is converted into a TF expression of the desired form using the appropriate TF of the fractional controller.

4. Given the identity of the characteristic polynomials of the studied control loop and the selected desired fractional form, an equation will be obtained on the basis of which the TF of the fractional controller is synthesized.

For example, the general four-loop block diagram of the SSR of a two-mass electromechanical system is considered, shown in Fig. 1. The basis of this electric drive is the «thyristor converter – motor» (TC-M) electric drive.

A common algorithm for synthesizing multi-loop SSR with cascade connection of controllers is the alternate synthesis of controllers, starting exactly from the inner loop and gradually moving to the outer loop. Each subsequent controller is synthesized after the synthesis of the internal loop is complete. Thus, we will first consider the DC motor armature current control loop under the following condition – internal EMF feedback. The DC motor can be neglected.

5. 2. Results of synthesis of armature current and DC motor speed controllers

5. 2. 1. Results of the synthesis of the armature current controller using the desired integer- and fractional-order forms

According to the block diagram shown in Fig. 1, the TF of the closed current loop of the electric drive $W_I(s)$ is as follows:

$$W_I(s)=\frac{W_{RI}(s)\dfrac{K_{TP}}{(T_{TP}s+1)}\dfrac{1/R_a}{(T_as+1)}}{1+W_{RI}(s)\dfrac{K_{TP}}{(T_{TP}s+1)}\dfrac{1/R_a}{(T_as+1)}K_{Ia}}. \qquad (2)$$

Option 1.

Further, dividing the numerator and denominator of the obtained TF by its numerator, and immediately taking into account that the expression of the desired characteristic polynomial is, for example, of the first order, we get:

$$W_I(s)=\frac{1}{\dfrac{(T_{TP}s+1)(T_as+1)R_a}{W_{RI}(s)K_{TP}s}s+K_{Ia}}. \qquad (3)$$

Let's set, for example, the desired integer standard form: binomial or first-order Butterworth with a known TF:

$$W_{sI}(s)=\frac{1/K_{Ia}}{T_{\mu I}s+1}, \qquad (4)$$

where $T_{\mu I}$ is the small uncompensated time constant of the DC motor armature current control loop, which takes into account the time constant of the thyristor converter T_{TP}, as well as the higher harmonic filter and inertia of the armature current measurement system ($T_{\mu I}\approx 2T_{TP}$).

The ratio of time constants $T_{\mu I}<T_a$ means that the speed of the synthesized DC motor current loop will be determined only by the small time constant.

Now let's set the requirement to transform the obtained expression (3) into expression (4).

Further, given the identity of the characteristic polynomials $H_{sI}(s)$ and $H_I(s)$, we get:

$$\frac{(T_{TP}s+1)(T_as+1)R_a}{2W_{RI}(s)K_{TP}K_{Ia}T_{TP}s}=1. \qquad (5)$$

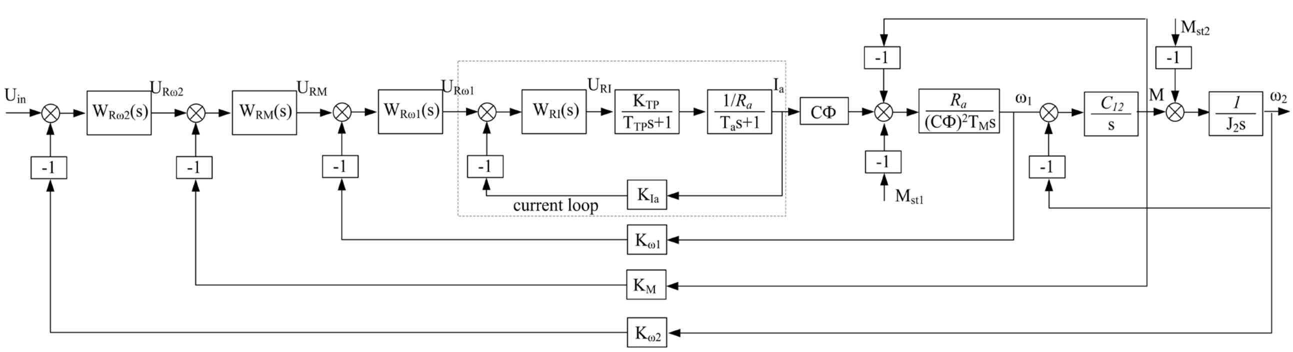

Fig. 1. General block diagram of a four-loop subordinate regulation system of the two-mass system with the following notations: $W_{R\omega1}(s)$, $W_{R\omega2}(s)$ – TF of the motor and actuator speed controllers; $K_{\omega1}$, $K_{\omega2}$ – feedback factors for the speeds of both moving masses; $W_{RI}(s)$ – TF of the current controller; $W_M(s)$ – TF of the torque controller; K_{Ia} – current feedback factor; K_M – torque feedback factor; and the rest of the designations are generally accepted: T_{TP} – TC time constant; K_{TP} – TC gain factor; C – motor design constant; R_a – total resistance of the armature circuit; T_a – electromagnetic time constant of the armature circuit; T_M – electromechanical time constant of the electric drive; M_{st} – static load moment; Φ – magnetic excitation flux

Then we found:

$$W_{RI}(s)=\frac{(T_{TP}s+1)(T_a s+1)R_a}{2K_{TP}K_{Ia}T_{TP}s}. \quad (6)$$

Substituting the numerical parameters of the links included in the DC motor current loop, we obtain the following TF of the current controller:

$$W_{RI}(s)=1.22+\frac{22.896}{s}+0.00378s. \quad (7)$$

Thus, as expected, an integer PID controller is obtained for the integer standard form. Fig. 2 shows the corresponding armature current transition function found as the response of the internal current loop to the spiking effect of the task for this loop.

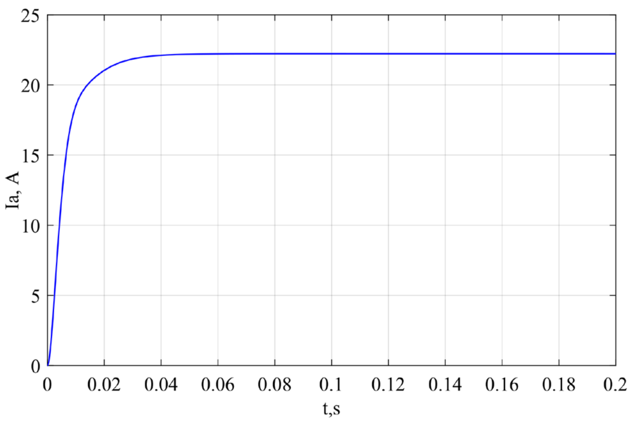

Fig. 2. Transition function of the optimized loop with the integer current PID controller

Option 2.

The current controller will now be synthesized in a similar way, based on the desired fractional form $W_{s1}(s)$ (1).

Similarly, the requirement is set to transform the obtained expression (3) into expression (1), to which the current feedback factor $K_1=K_{Ia}$ was introduced.

Similarly, given the identity of $W_I(s)$ and $W_{s1}(s)$, we get:

$$\frac{1}{\dfrac{(T_{TP}s+1)(T_a s+1)R_a}{W_{RI}(s)K_{TP}}+K_{Ia}}=\frac{\omega_{o1}/K_{Ia}}{s^q+\omega_{o1}}. \quad (8)$$

Solving equation (8), we get:

$$W_{RI}(s)=\frac{(T_{TP}s+1)(T_a s+1)R_a\omega_{o1}}{K_{TP}K_{Ia}s^q}. \quad (9)$$

Thus, as a result of the synthesis, the structure of the fractional-order DC motor armature current controller is obtained.

Now, as an example, let's set the desired dynamic characteristics for the armature current: overshoot δ=6.8 % and time to reach 95 % of the established value $t_{0.95}$=0.042 s. This is ensured by the desired fractional-order form (line No. 4 in Table 1) with parameters q=1.2, ω_{o1}=100 s^{-1}.

Table 1

Parameters of transition functions for the desired fractional-order form (1) for ω_{o1}=100 s^{-1}

No.	q	δ, %	t, s	t_{reg}, s
1	0.9	0	0.0299	0.0299
2	1.0	0	0.0319	0.0319
3	1.1	2.5	0.0361	0.0361
4	1.2	6.76	0.0424	0.1106
5	1.3	11.52	0.0506	0.1628

Next, by substituting the numerical parameters of the component links into the DC motor current loop, we will obtain the resulting TF of the current controller for ω_{o1}=100 s^{-1}:

$$W_{RI}(s)=\frac{0.805}{s^{0.2}}+\frac{15.111}{s^{1.2}}+0.0025s^{0.8}. \quad (10)$$

Research for this synthesis case, as well as all other options, was carried out using simulation models implemented in the MATLAB Simulink package and additional NINTEGER package.

Using the synthesized controller (10), we obtain a transition process with the following parameters: δ=7.0 %, $t_{0.95}$=0.0404 s (Fig. 3, curve 3), that is, the analysis of the deviation of the results from the given parameters does not exceed 1 %.

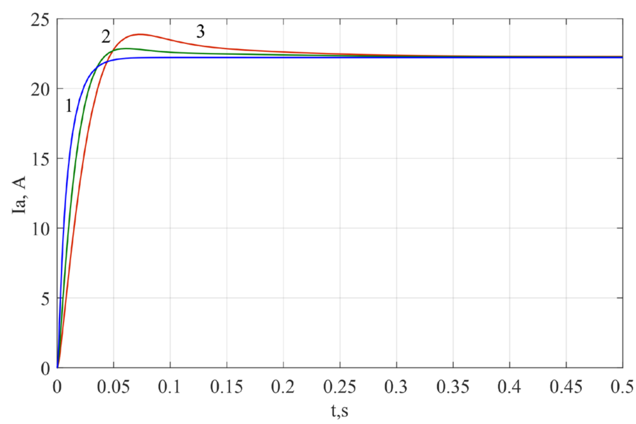

Fig. 3. Transition functions of the optimized DC motor current loop with the current controller (9), if: q=1.0 – curve 1; q=1.1 – curve 2; q=1.2 – curve 3

To demonstrate the capabilities of the approach, by substituting the EMS parameters given (q=1.0–1.1 for ω_{o1}=100 s^{-1}), we obtain the following expressions for the TF of the current controller:

$$W_{RI}(s)=0.805+\frac{15.111}{s}+0.0025s \text{ for } q=1.0,$$

$$W_{RI}(s)=\frac{0.805}{s^{0.1}}+\frac{15.111}{s^{1.1}}+0.0025s^{0.9} \text{ for } q=1.1.$$

According to the results of studies of the transition functions of the DC motor armature current provided that q varies from 1.0 to 1.2 for ω_{o1}=100 s^{-1}, graphs were obtained, which are shown in Fig. 3.

Thus, this approach allowed the synthesis of a current controller, provided that both the integer standard form and the proposed desired fractional-order form are used.

5. 2. 2. Results of the synthesis of the DC motor speed controller using the desired fractional-order form

At the next stage, the synthesis procedure of the first angular velocity controller of the two-mass electric drive is considered, taking into account the previous results. The internal current loop was optimized using either the standard integer form or the desired fractional-order one.

Option 1.

To begin with, the first option for optimizing the speed loop of a two-mass electric drive is considered, provided that the DC motor current loop is synthesized in accordance with (4), i.e., an integer TF is obtained. Then the TF of the closed loop of the first speed of the two-mass electric drive, taking into account the TF of the optimized current loop (4), has the following form:

$$W_{\omega 1}(s)=\frac{W_{R\omega 1}(s)\dfrac{1/K_{Ia}}{T_{\mu I}s+1}\dfrac{R_a}{C\Phi T_M s}}{1+W_{R\omega 1}(p)\dfrac{1/K_{Ia}}{T_{\mu I}s+1}\dfrac{R_a}{C\Phi T_M s}K_{\omega 1}}. \quad (11)$$

Dividing the numerator and denominator of the obtained TF (11) by its numerator, we get:

$$W_{\omega 1}(s)=\frac{1}{\dfrac{K_{Ia}(T_{\mu I}s+1)C\Phi T_M s}{W_{R\omega 1}(s)R_a}+K_{\omega 1}}. \quad (12)$$

Now let's assume the desired fractional form (1). Then for the first speed loop, the following is written:

$$W_{s\omega 1}(s)=\frac{\omega_{o\omega 1}/K_{\omega 1}}{s^q+\omega_{o\omega 1}}, \quad (13)$$

where $\omega_{o\omega 1}$ is the value of the geometric mean root of the first speed loop.

Given the equality of expressions $W_{\omega}(s)=W_{s1}(s)$, we obtain the following expression of the TF of the first speed controller:

$$W_{R\omega 1}(s)=\frac{(T_{\mu I}s+1)C\Phi T_M K_{Ia}\omega_{o\omega 1}s}{R_a K_{\omega 1}s^q}. \quad (14)$$

By substituting the EMS parameters given ($q=1.0$–1.2 for $\omega_{o\omega 1}=50\ s^{-1}$), the following TF of the speed controller was obtained:

$$W_{R\omega 1}(s)=44+0.2904s \text{ for } q=1.0,$$

$$W_{R\omega 1}(s)=\frac{44}{s^{0.1}}+0.2904s^{0.9} \text{ for } q=1.1,$$

$$W_{R\omega 1}(s)=\frac{44}{s^{0.2}}+0.2904s^{0.8} \text{ for } q=1.2.$$

After examining the obtained transition functions of the first motor speed, we obtain a family of graphs, which are shown in Fig. 4.

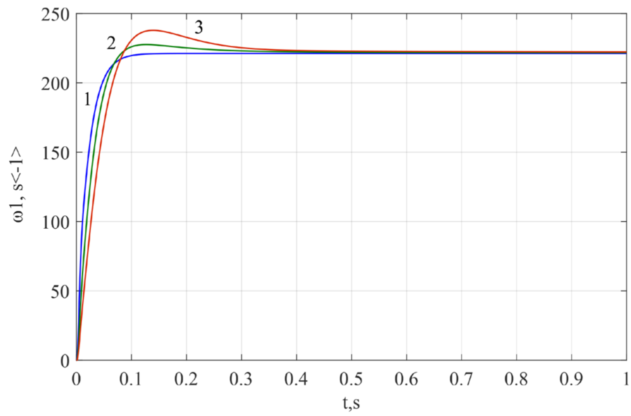

Fig. 4. Transition functions of the optimized motor speed loop with the speed controller (14), if: $q=1.0$ – curve 3; $q=1.1$ – curve 4; $q=1.2$ – curve 5

Option 2.

Similarly to the current loop, the case of optimizing the first speed loop of the two-mass electric drive is considered, provided that the current loop was synthesized according to (1), that is, the fractional TF of the current loop was obtained.

In this case, the following expression of the TF of the first speed loop is obtained:

$$W_{\omega 1}(s)=\frac{W_{R\omega}(s)\dfrac{\omega_{oI}/K_{Ia}}{s^q+\omega_{oI}}\dfrac{R_a}{C\Phi T_M s}}{1+W_{R\omega}(s)\dfrac{\omega_{oI}/K_{Ia}}{s^q+\omega_{oI}}\dfrac{R_a}{C\Phi T_M s}K_{\omega 1}}. \quad (15)$$

Dividing the numerator and denominator of the obtained TF (15) by its numerator, we get:

$$W_{\omega 1}(s)=\frac{1}{\dfrac{(s^q+\omega_{oI})C\Phi T_M K_{Ia}s}{W_{R\omega}(s)\omega_{oI}R_a}+K_{\omega 1}}. \quad (16)$$

As in the previous case, let's set the desired fractional form (13). In this case, given the equality $W_{\omega}(s)=W_{s\omega 1}(s)$, we get:

$$W_{R\omega 1}(s)=\frac{(s^q+\omega_{oI})C\Phi T_M K_{Ia}\omega_{o\omega 1}s}{R_a\omega_{oI}K_{\omega 1}s^q}. \quad (17)$$

By substituting the EMS parameters given ($q=1.0$–1.2 for $\omega_{o\omega 1}=50\ s^{-1}$), the following expressions for the TF of the first speed controller were obtained:

$$W_{R\omega 1}(s)=44+0.44s \text{ for } q=1.0,$$

$$W_{R\omega 1}(s)=\frac{44}{s^{0.1}}+0.44s^{0.9} \text{ for } q=1.1,$$

$$W_{R\omega 1}(s)=\frac{44}{s^{0.2}}+0.44s^{0.8} \text{ for } q=1.2.$$

The transition processes of the first speed in the currently synthesized two-loop first speed SSR are similar to the results shown in Fig. 5. This can be explained by the fact that all obtained expressions of the TF of the first speed controllers are obtained by applying the same desired fractional TF.

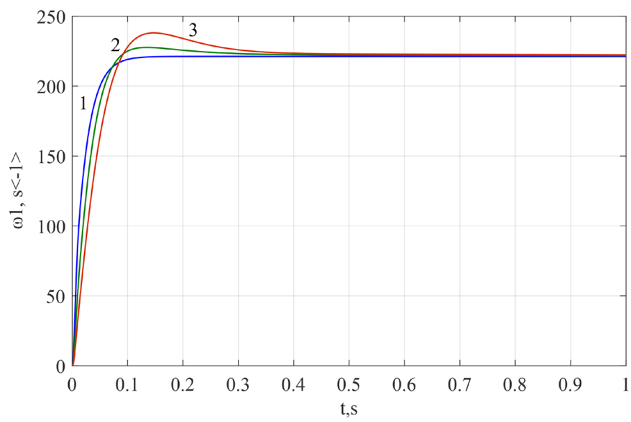

Fig. 5. Transition functions of the optimized motor speed loop with the speed controller (17), if: q=1.0 – curve 1; q=1.1 – curve 2; q=1.2 – curve 3

5. 3. Results of the synthesis of torque and second angular velocity controllers of the mechanism

5. 3. 1. Results of the synthesis of the torque controller using the desired fractional-order form

At the next stage, the procedure for synthesizing a two-mass electric drive torque controller is considered, taking into account the previous results, namely that the internal first speed current loop of the two-mass electric drive was optimized provided that the desired fractional-order form is used.

An option of optimizing the torque loop of the two-mass electric drive is considered, provided that the first speed current loop of the DC motor was synthesized according to (1), that is, a fractional TF of the current loop was obtained. Then the transfer function of the closed torque loop of the two-mass electric drive, taking into account the TF of the optimized loop (13), will be as follows:

$$W_M(s)=\frac{W_{RM}(s)\dfrac{\omega_{o\omega1}/K_{\omega1}}{s^q+\omega_{o\omega1}}\dfrac{J_1CC_{12}s}{J_1Cs^2+C_{12}}}{1+W_{RM}(s)\dfrac{\omega_{o\omega1}/K_{\omega1}}{s^q+\omega_{o\omega1}}\dfrac{J_1CC_{12}s}{J_1Cs^2+C_{12}}K_M}. \quad (18)$$

Dividing the numerator and denominator of the obtained TF (18) by its numerator, we get:

$$W_M(s)=\frac{1}{\dfrac{(s^q+\omega_{o\omega1})(J_1Cs^2+C_{12})K_{\omega1}}{W_{RM}(s)\omega_{o\omega1}J_1CC_{12}s}+K_M}. \quad (19)$$

Now let's assume the desired fractional form (1). Then for the torque loop, we get:

$$W_{sM}(s)=\frac{\omega_{oM}/K_M}{s^q+\omega_{oM}}, \quad (20)$$

where ω_{oM} is the value of the geometric mean root of the first speed torque loop.

Given the equality of expressions $W_M(s)=W_{sM}(s)$, the following expression of the TF of the torque controller is obtained:

$$W_{RM}(s)=\frac{K_{\omega1}(s^q+\omega_{o\omega1})(J_1Cs^2+C_{12})\omega_{oM}}{\omega_{o\omega1}J_1C_{12}K_MCss^q}. \quad (21)$$

By substituting the EMC parameters given (q=1.0–1.2 for ω_{oM}=25 s^{-1}), the following TF of the torque controller was obtained:

$$W_{RM}(s)=0.000225s^{1.2}+\frac{0.08182}{s^{0.8}}+0.01125+\frac{4.09091}{s^2}$$

for q=1.0,

$$W_{RM}(s)=0.000225s^{1.1}+\frac{0.08182}{s^{0.9}}+\frac{0.01125}{s^{0.1}}+\frac{4.09091}{s^{2.1}}$$

for q=1.1,

$$W_{RM}(s)=0.000225s+\frac{0.08182}{s}+\frac{0.01125}{s^{0.2}}+\frac{4.09091}{s^{2.2}}$$

for q=1.2.

After examining the resulting transition functions of the motor torque, a family of graphs was obtained, which are shown in Fig. 6.

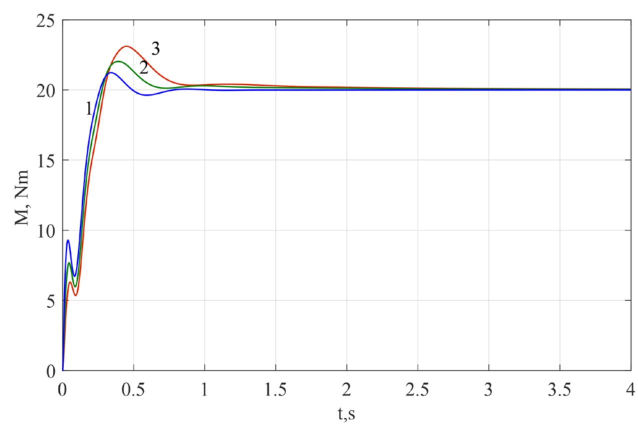

Fig. 6. Transition functions of the optimized speed torque loop with the torque controller (21), if: q=1.0 – curve 1; q=1.1 – curve 2; q=1.2 – curve 3

The type of torque transitions in the synthesized four-loop SSR corresponds to the desired fractional TF applied.

5. 3. 2. Results of the synthesis of the second angular velocity controller using the desired fractional-order form

At the final stage, the procedure for synthesizing the second angular velocity of the two-mass electric drive was considered, taking into account the previous results, namely that the internal torque current loop of the two-mass electric drive was optimized provided that the desired fractional-order form is used.

Optimization of the second angular velocity torque loop of the two-mass electric drive is considered, provided that the torque loop was synthesized according to (1), that is, the fractional TF of the torque loop was obtained. Then the transfer function of the closed second angular velocity loop of the two-mass electric drive, taking into account the TF of the optimized loop (20), will be as follows:

$$W_{\omega2}(s)=\frac{W_{R\omega2}(s)\dfrac{\omega_{oM}/K_M}{s^q+\omega_{oM}}\dfrac{s}{J_2s^2+C_{12}}}{1+W_{R\omega2}(s)\dfrac{\omega_{oM}/K_M}{s^q+\omega_{oM}}\dfrac{s}{J_2s^2+C_{12}}K_{\omega2}}. \quad (22)$$

Dividing the numerator and denominator of the obtained TF (22) by its numerator, we get:

$$W_{\omega 2}(s)=\frac{1}{\dfrac{(s^q+\omega_{oM})(J_2 s^2+C_{12})K_M}{W_{R\omega 2}(s)\omega_{oM}s}+K_{\omega 2}}. \quad (23)$$

Now let's assume the desired fractional form (1). Then for the second angular velocity loop, we get:

$$W_{s\omega 2}(s)=\frac{\omega_{o\omega 2}/K_{\omega 2}}{s^q+\omega_{o\omega 2}}, \quad (24)$$

where $\omega_{o\omega 2}$ is the value of the geometric mean root of the second speed torque loop.

Given the equality of expressions $W_{\omega 2}(s)=W_{s1}(s)$, the following expression of the TF of the second angular velocity controller is obtained:

$$W_{R\omega 2}(s)=\frac{K_M(s^q+\omega_{oM})(J_2 s^2+C_{12})\omega_{o\omega 2}}{\omega_{oM}K_{\omega 2}ss^q}. \quad (25)$$

By substituting the EMS parameters given (q=1.0–1.2 for $\omega_{o\omega}$=10 s^{-1}), the parameters of the transition process will be close to the desired fractional-order form in Table 2 for ω_o=10 s^{-1}, the following TF of the second speed controller is obtained:

$$W_{R\omega 2}(s)=1.222s^{1.2}+\frac{1111.1}{s^{0.8}}+30.556+\frac{27778}{s^2} \text{ for } q=1.0,$$

$$W_{R\omega 2}(s)=1.222s^{1.1}+\frac{1111.1}{s^{0.9}}+\frac{30.556}{s^{0.1}}+\frac{27778}{s^{2.1}} \text{ for } q=1.1,$$

$$W_{R\omega 2}(s)=1.222s+\frac{1111.1}{s}+\frac{30.556}{s^{0.2}}+\frac{27778}{s^{2.2}} \text{ for } q=1.2.$$

After examining the obtained transition functions of the first and second speeds of the two-mass electric drive, we get the graphs shown in Fig. 7.

Table 2

Parameters of transition functions of the desired fractional-order form (1) for ω_{o1}=10 s^{-1}

No.	q	δ, %	t, s	t_{reg}, s
1	0.9	0	0.365	0.365
2	1.0	0	0.3	0.3
3	1.1	2.7	0.28	0.28
4	1.2	7.3	0.28	0.75
5	1.3	13.3	0.29	0.94

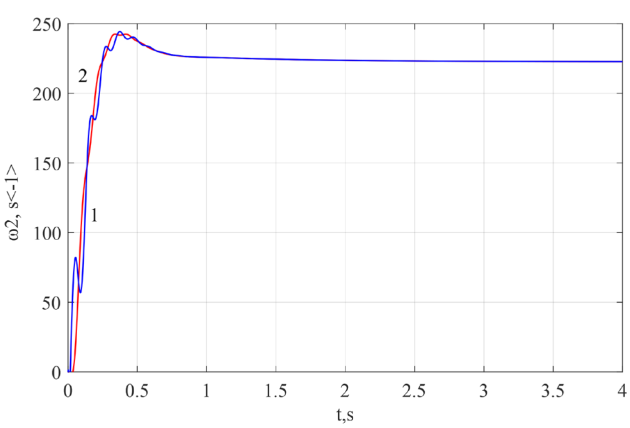

Fig. 7. Transition functions of the first (curve 1) and second (curve 2) speed of the two-mass electric drive

The use of the desired fractional form made it possible to obtain the transition processes of the first and second speeds of the two-mass EMS with overshoot and rise time, which were previously specified.

6. Discussion of the results of developing a method of structural-parametric synthesis of fractional-order controllers

The proposed approach to the synthesis of ACS loops using the desired fractional form made it possible to ensure the necessary quality of the transition process, in particular, the desired fractional form allows changing both the speed and overshoot.

In contrast to the works [13, 14], where a single-loop automatic control system was considered, this paper considered a four-loop ACS, which made it possible to alternately adjust the loops and ensure the necessary dynamic indicators of each loop. This becomes possible due to the modernization of the GCP synthesis method for two-mass EMSs using fractional-order TFs. This allowed using the desired fractional-order form. Unlike the work [11], where only parametric synthesis is considered, this paper examines structural-parametric synthesis. This made it possible to alternately calculate, rather than search by the selection method, the parameters of fractional controllers of each loop. All this became possible due to the modernization of the GCP synthesis method for fractional-order EMS and, as a result, the appearance of the desired fractional-order form.

The internal motor current loop was synthesized both using the integer standard form, i.e. the classical method, and using the desired fractional form. The desired fractional form allowed for the desired overshoot and speed, which cannot be provided by the standard integer form. In Fig. 3 curve 1, the following was obtained: δ=0 % and $t_{0.95}$=0.0304 s (given in Table 1 for q=1.0: δ=0 % and $t_{0.95}$=0.0319 s), curve 2 δ=2.7 % and $t_{0.95}$=0.0332 s (given in Table 1 for q=1.1: δ=2.5 % and $t_{0.95}$=0.0361 s), curve 3 δ=7.1 % and $t_{0.95}$=0.0404 s (given in Table 1 for q=1.2: δ=6.76 % and $t_{0.95}$=0.0424 s). The error of the obtained results, compared to the specified ones, is no more than 8.5 %.

The first motor speed loop was synthesized based on the results obtained above, that is, the desired fractional form was applied for both synthesis options of the current loop, and similarly they provided both the desired overshoot and speed. A very important point is observed. It was possible to combine both integer standard forms and the desired fractional forms. In Fig. 4 curve 1, the following was obtained: δ=0 % (set for q=1.0: δ=0 %), curve 2 δ=2.62 % (set for q=1.1: δ=2.5 %) and curve 3 δ=7.02 % (set for q=1.2: δ=6.76). The error of the obtained results is no more than 5 %.

The torque and second speed loops were synthesized using only the desired fractional form. The results in Fig. 7 showed the full implementation of the initial synthesis plan by the proposed modified method – the necessary overshoot and coordinate rise time were provided. The desired fractional form provides an overshoot to a relatively simple form of TF, but in this case, the integer form would have to be taken of a higher order. In this case, the synthesis process would be significantly complicated and, of course, certain additional assumptions would have to be made. An analysis of the obtained transition processes was carried out.

In Fig. 5 curve 1, the following was obtained: δ=0 % (set for q=1.0: δ=0 %), curve 2 δ=2.62 % (set for q=1.1: δ=2.5 %) and curve 3 δ=7.02 % (set for q=1.2: δ=6.76 %). The error of the obtained results is no more than 5 %.

In Fig. 7 curve 1, the following was obtained: δ=9.6 % and $t_{0.95}$=0.237 s, curve 2 δ=8.87 % and $t_{0.95}$=0.213 s (given in Table 2 for q=1.2: δ=7.3 % and $t_{0.95}$=0.28 s).

Transition processes of the first and second speed in the synthesized four-loop SSR were obtained by applying the desired fractional TF. These results confirmed the possibility of applying the proposed original approach to the synthesis of two-mass EMS ACS loops based on the fractional characteristic polynomial. The fractional characteristic polynomial ensured, during the synthesis, the desired quality of the transition process in the case of implementing the defined structure of the fractional controller.

The technical implementation of fractional regulators on various controllers was carried out [17] in 2016 and does not cause difficulties for further implementation in real two-mass EMSs. The limitations inherent in this study include the fact that a fractional-order controller can only have a digital implementation on a high-performance microcontroller. Analog implementation of fractional controllers is impractical. The studies were carried out taking into account the previous experience of the actual use of fractional-order controllers in single-mass EMSs on a laboratory bench.

The limitations of the studies include the following: only two-mass EMSs are considered, the general block diagram of which either fully corresponds or is reduced by simplification to the block diagram shown in Fig. 1. This statement also applies to the scope of the proposed synthesis approach.

The application of the proposed desired fractional forms greatly expands the range of possible settings of fractional-order controllers in the process of synthesis of EMS loops. This approach provides a better quality of transition processes compared to integer-order controllers, and thus increases the efficiency of synthesized EMSs.

The proposed approach to the synthesis of ACS loops using the desired fractional form ensures the necessary quality of the transition process. The desired fractional form allows for changes in both speed and overshoot, which cannot be provided by the integer standard form.

Structural-parametric synthesis of fractional-order controllers for two-mass EMS with cascade connection of controllers confirmed its effectiveness. The research results demonstrated the possibility of using cascaded controllers for two-mass EMS with the TF of integer- and fractional-order controllers, as well as systems with only fractional-order controllers.

The shortcomings of the conducted research include the fact that the linear model of a thyristor converter and DC motor of independent excitation was used.

The development of this research may consist in developing such a model based on the use of elements from the Simscape MATLAB library and implementing a laboratory bench of a two-mass EMS for research.

7. Conclusions

1. The paper highlights the modernization of the GCP synthesis method for two-mass EMSs using a fractional-order TF and developed the corresponding algorithm for the synthesis of fractional-order controllers for the necessary control loops of a two-mass electromechanical system. The proposed modernization consists in the structural-parametric search for the TF of fractional-order controllers based on the formation and solution of an equation for a given desired fractional-order form.

2. In the work, the current and first speed controllers of a four-loop SSR were synthesized by using both the standard integer form and the desired fractional form, which are characterized by a given wide range of dynamic properties and ensured both the desired overshoot and speed. A very important point was noted. It was possible to combine both integer standard forms and the desired fractional forms.

3. In the work, torque and second speed controllers of a four-loop SSR were synthesized using only the desired fractional form. The results showed the full implementation of the initial synthesis plan by the proposed modified method – the required overshoot and coordinate rise time were ensured. For the transition function of the motor speed, the following parameters of the transition process were obtained: δ=9.6 %, $t_{0.95}$=0.237 s; for the second speed of the two-mass electric drive, δ=8.87 %, $t_{0.95}$=0.213 s. In addition, it is shown that an increase in the q value of the desired fractional form leads to an increase in the magnitude of the coordinate overshoot during the processing of the task signal. Based on the analysis of the dynamic properties of two-mass EMSs, the possibility of two implementations of cascaded controllers for electromechanical systems is demonstrated. In the first option, loops with integer- and fractional-order TF are combined, in the second – only fractional-order ones.

Conflict of interest

The authors of this paper have no conflict of interest in relation to this study, whether financial, personal, authorship or other.

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The study was conducted without financial support.

Data availability

Data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the presented work.

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