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The genetic algorithm for transition from high to fractional order controllers of a two-mass positional electromechanical system

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ABSTRACT

A new original approach to the synthesis of the loops of automatic control systems of two-mass positional electromechanical systems is proposed in the article based on the application of the generalized characteristic polynomial at the first stage and the intelligent optimization method at the second. In practice, the skylifting mechanism of a fire truck is a complex control object. Imperfect manufacturing of mechanical components and their connections, elastic deformations of the boom during operation and supply of fire-extinguishing substance cause rescue cage oscillations. The use of an automatic control system makes it possible to damp the elastic boom vibrations. The synthesized automatic control system, which controls the movement of the boom, must meet the following requirements: the necessary speed, static and dynamic accuracy of the rescue cage movement, the absence of significant adjustments in transient modes, etc. To meet these requirements, an analysis of various automatic control systems and methods of their synthesis was carried out. As a result of the analysis, a two-mass positional three-loop system of subordinate regulation by the rescue cage rotation mechanism, taking into account the elastic properties of the boom was created using the generalized characteristic polynomial method. The synthesized system of subordinate regulation allows damping of elastic oscillations, providing the desired transition processes of the rescue cage rotation mechanism and low sensitivity in the stable mode to the action of disturbances. The transfer functions of the angular speed controllers of the motor and rescue cage obtained in the process of synthesis are high order and turned out to be quite complex from the point of view of practical implementation. It is proposed to replace these controllers with more compact fractional order controllers. The conducted research using mathematical modelling confirmed the effectiveness of replacing high-order controllers of the angular speed of the motor and rescue cage with fractional order controllers. The transfer functions of these controllers are determined by approximating the transfer functions of the controllers using a genetic algorithm.

Keywords: Fire skylift; two-mass electromechanical system; original controller’s synthesis; transition to fractional order controllers; genetic algorithm

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INTRODUCTION

For rescue operations and firefighting at significant heights, various types of specialized a fire truck ladders equipped with or without rescue cage, as well as fire skylift are used. The work of rescuers using such equipment is dangerous both for them and for potentially rescued persons. Therefore, high requirements are put forward to the skylifting mechanism [1, 2].

Structurally, the rescue cage movement system consists of the following main mechanisms: platform rotation, raising/lowering and expanding/ folding the boom (hinged mechanism) for articulated skylifts or extension/folding for telescopicskylifts. Each of them has its own control system, and in modern skylift truck they are united by a common control system at the highest hierarchical level. The quality of the transient process of moving the rescue cage depends on the operation of all these control systems.

Since the boom of the skylifting mechanism is not completely rigid, deformations of the boom occur. Due to elastic deformations of the boom, imperfect manufacturing of mechanical elements and their connections, large masses of moving parts, inefficient operation of the control system itself, reactive action of fire extinguishing means, wind load when movement of the rescue cage, oscillations occur in both the vertical and horizontal planes, which are called "coordinate perturbations". These factors negatively affect the operation of the system and complicate the rescuers work.

Damping of oscillations with only mechanical devices is ineffective. Therefore, along with mechanical devices [3], a promising way to reduce or eliminate oscillations is the use of an automatic control system (ACS) [4], which makes the rescuers work safer and more efficient.

The ACS should function in positioning and stabilization modes. Parametric and coordinate perturbations are possible in these modes. Parametric disturbances a include changes in the inertia moment of the second mass due to a changes in the position of the rescue cage and parameters of electrical equipment due to a changes in environmental climatic conditions and physical "aging". Coordinate disturbances are caused by the reactive effect of fire-extinguishing substance and wind load.

The ACS of the rescue cage rotation of the fire skylift must have minimal sensitivity to all these disturbances. It should be configured to ensure a monotonic of the angular coordinate transition process in the ideal case. The desire to improve dynamic and static indicators requires the ACS of the drive of the rescue cage rotation mechanism to perform more complex tasks. Such an ACS should ensure: the required speed, static and dynamic accuracy of the rescue cage movement, absence of over-regulation in transient modes, etc.

LITERATURE REVIEW

One of the main control coordinates is the angle of rescue cage rotation. However, the structure of the ACS by the rescue cage rotation mechanism includes an internal loop for controlling the speed of the output shaft of the executive motor and an internal loop for controlling the rescue cage rotation speed. Quantitative and qualitative indicators of the rescue cage rotation angular coordinate depend significantly on their settings.

In order to ensure the necessary indicators of the angular coordinates of the rescue cage rotation, a number of fundamentally different ACSs and methods of their synthesis were analyzed. It is possible to ensure a monotonous transition process with the help of modal control [5, 6]. When synthesizing a modal control system (MCS), an important aspect of problem solving is the choice of desired standard characteristic polynomials. However, despite the significant advantages of such a system, it is difficult to configure and does not provide the ability to adjust intermediate coordinates, but only the original one.

Such shortcomings can be eliminated by building ACS based on the principle of subordinate regulation [6, 7], [8]. Traditionally, the system of subordinate regulation (SSR) of the electric drive coordinates is adjusted on the basis of two standard forms of the distribution of the roots of the characteristic equation, namely: according to the modular (technical) optimum and according to the symmetrical optimum. However, problems arise in ensuring static and dynamic accuracy, as well as sensitivity to parametric disturbances. In addition, the formation of the dynamic characteristics of ACS is affected by the location on the complex plane of the poles and zeros of their transfer functions (TF).

The SSR during synthesis for symmetric or modular optimum and MCS do not allow compensating zeros of TF. The TF zeros of electromechanical systems (EMS) are compensated with the help of appropriate filters connected to the system input [6]. It is clear that in this case it is possible to obtain the entire range of standard forms of transient functions, if there is no problem related with a decrease of the speed and the limit of stability in terms of the system amplitude. Thus, in a SSR configured for a symmetrical optimum, the introduction of a filter at the system input leads to a decrease in speed.

High ACS indicators can be obtained using the principle of combined control [6]. However, the disadvantage of this approach is that several synthesis methods are considered when considering one positional SSR.

An alternative version of the construction of the ACS of the rescue cage rotation mechanism is the SSR, synthesized by the method of the generalized characteristic polynomial (GCP) [9]. This method allows it possible to take into account the zeros of the transfer function during the synthesis of the control system and adjust it to any standard form of the transition process of the output controlled coordinate. In addition, this method ensures the stability of the system against disturbances [9].

On the basis of the research conducted in [10, 11], [12, 13], it is shown that fractional order (FO) controllers have advantages over classical ones, in particular, in the case of their use for the optimization of two-mass EMSs.

In article [10], the application of the Caputo–Fabrizio operator to improve the mathematical model of a two-mass system with a long shaft and concentrated parameters is analyzed. The approach proposed in the article makes it possible to develop a mathematical model of two-mass systems using a fractional integrator. This mathematical model allows you to describe real transient processes. It can be used to build a mathematical model of a two-mass turning mechanism of a fire truck. However, the article does not consider the synthesis of controllers for this two-mass system.

In article [13], a fractional $PI^{\lambda}D^{\mu}$-controller and an active disturbance elimination controller for a nonlinear two-mass system were developed. However, it is important to note that the $PI^{\lambda}D^{\mu}$-controller was designed with feedback based on the first mass speed. In addition, the active perturbation controller used an observer that provided information about the control coordinates.

In article [14], various optimization algorithms of FO $PI^{\lambda}D^{\mu}$-controllers are analyzed. Each optimization algorithm has different performance not only due to the structure of the algorithm, but also depending on the optimization problem. It is proved that for a two-wheeled inverted pendulum, the best result is achieved by the FO controller, which is optimized using the artificial bee swarm algorithm. In article [15], a FO $PI^{\lambda}D^{\mu}$-controller for the rotation angle of the fire control system of the gun barrel was synthesized. To solve the problem of nonlinear multi-criteria optimization, a new multi-criteria differential evolution algorithm based on the Pareto optimal solution is proposed. In the work, the mathematical model of the gun barrel rotation mechanism is presented as a one-mass one, taking into account external viscous friction.

In article [16], the method of synthesis using GCP for multi-loop EMSs under the condition of cascade activation use of FO controllers was improved. The algorithm for the synthesis of FO controllers of the corresponding control circuits is presented. The conducted studies showed the possibility of introducing cascaded fractional controllers for EMSs, where loop with TFs of integer and FO are combined, as well as systems with loop of the only FO. However, in the work, an SSR with single-mass and two-loops was synthesized, where the object of regulation is the electric drive “thyristor converter – motor”.

In article [17], the authors focused on the synthesis of a fractional controller for a wind power plant using a multi-criteria genetic algorithm. The mathematical model of the wind power plant was considered as a two-mass system with integral integrators. The authors showed that this fractional controller outperforms other controllers in terms of overall efficiency. It is important to note that in this work the system was constructed as a single-loop system.

THE PURPOSE OF THE ARTICLE

In this article, for the damping of elastic vibrations in the horizontal plane, it is proposed to synthesize the positional SSR by the rescue cage rotation mechanism, taking into account the elastic properties of the boom. This will ensure the necessary dynamic and static characteristics of the rescue cage movement under the influence of control and disturbance actions on it. Thus, increase the efficiency of rescuers work in real conditions. To implement the obtained controllers for the angular speed of the motor and rescue cage, it is suggested to replace them by approximation by evolutionary methods with more compact $PI^{\lambda}D^{\mu}$-controllers or FO controllers.

To achieve the stated goal, the following scientific tasks must be completed:

– to synthesize a SSR by the rescue cage rotation mechanism taking into account the elastic properties of the boom;

– to obtain the controllers of the angular speed of the motor and rescue cage, replacing them by approximation by evolutionary methods with more compact $PI^{\lambda}D^{\mu}$-controllers or FO controllers.

SYNTHESIS OF TWO-MASS SYSTEM OF SUBORDINATE REGULATION BY THE RESCUE CAGE ROTATION MECHANISM

The process of SSR synthesis requires the creation of a mathematical model of the control object: the executive motor and the rescue cage rotation mechanism, taking into account the influence of elastic deformations of the boom. In article [18], to improve dynamic and static indicators, it was shown the advantages of replacing the mechanical gear rotation the platform system of a fire truck with an electromechanical gearless one, which is driven by a torque valve motor (TVM). The TVM provides high accuracy and a large moment on the shaft at low speeds. The absence of a reducer in such a drive makes it possible to simplify the mechanical part of the drive: get rid of backlash and eliminate the problem of dynamic shocks during start and stop, thereby increasing the rigidity of the rotation mechanism. In article [18] it is stated that the nominal rotation frequency of the lifting-rotation mechanism is approximately 2 r/min, then the angular speed of rotation ω = 0.21 rad/s. In article [19], an motor with the following rated parameters was used for TVM simulation for the platform rotation mechanism of the fire skylift: R_a = 1.52

Ohm; L_a = 0.0091 H; C = 131 Nm/A; M_{max} = 13000 Nm; M_n = 8750.8 Nm; ω_m = 0,37 rad/s; J_m = 2000 kg·m^2; U_n = 150 V. The electromagnetic time constant of such motors is small enough and is about 0.006 s (T_a = 0.006 s), and the time constant of the pulse-width converter T_c = 0.005 s. To synthesize the controllers for a small uncompensated time constant, the sum of the time constants of the pulse-width converter and the armature circuit
(T_μ = 0.011 s) was taken.

Since T_μ is much smaller than the electromechanical time constant of the electric drive ($T_{em} = JR_a / C^2$) and the moment of the second mass, in this case the TF of the electric part of the drive can be written:

$$W_{el.dr}(p) = K_{el} / (T_\mu p + 1). \qquad (1)$$

The developed mathematical model for the rotation mechanism of the rescue cage in the horizontal plane is based on the kinematic scheme of the variant of the two-mass system:

$$\left.\begin{array}{l} M_1(p) - a_1(p)\omega_1(p) - \left[M_{12}(p) - b_{12}\left(\omega_2(p) - \omega_1(p)\right)\right] = J_1 p \omega_1(p), \\ M_{12}(p) = \dfrac{C_{12}}{p}\left(\omega_1(p) - \omega_2(p)\right), \\ M_{12}(p) + b_{12}\left(\omega_1(p) - \omega_2(p)\right) \pm F_s(p) L = J_2 p \omega_2(p), \\ \varphi_c(p) = \dfrac{1}{p}\omega_2(p), \end{array}\right\} \qquad (2)$$

where $\omega_1(p)$ is angular speed of the motor; $\omega_2(p)$ is angular speed of the rescue cage; $\varphi_c(p)$ is angle of rescue cage rotation; M_1 is motor torque; $M_{12}(p)$ is moment of elastic deformation of the arrow in the horizontal plane; J_1, J_2 is total moment of inertia of the first and second masses; $F_s(p)$ is horizontal component of disturbances caused by the force of the wind and the reactive force from the water jet; C_{12} is elasticity coefficient of bending deformation; a_1 is coefficient of external viscous friction; b_{12} is coefficient of internal viscous friction in an elastic boom; p is Laplace operator.

In the case of using a gearless drive, the value of the coefficient of external viscous friction a_1 will be quite small, and the impact on the system will be minimal, so it can be neglected in the synthesis process.

Since it is quite problematic to implement feedback on the moment of elastic deformation of the boom, we will perform the synthesis of the SSR by the rescue cage rotation mechanism without taking into account the control loop for the moment of elastic deformation $M_{12}(p)$. Taking into account the accepted assumptions, the structural diagram of such an SSR, taking into account the elasticity properties of the boom, is shown in Fig. 1.

The following designations are used in the figure: $W_{cs1}(p)$, $W_{cs2}(p)$ are TFs of the angular speed controllers of the motor and rescue cage; $W_{cp}(p)$ is TF of the rescue cage position controller; K_{s1}, K_{s2}, K_p are transmission coefficients of the speed sensors of the first mass (motor), second mass (rescue cage) and position sensor.

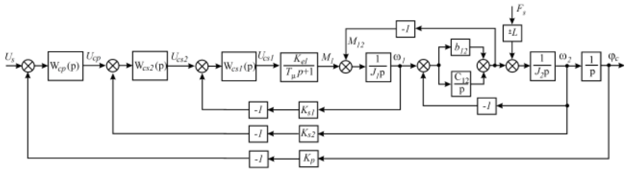

Fig. 1. **The structural diagram of the two-mass positional SSR by the rescue cage rotation mechanism**
Source: **compiled by the authors**

Let's consider three options for the synthesis of a two-mass positional SIR by the rescue cage rotation mechanism using the GCP method. Previous studies have shown that the decomposition version of the synthesis using the GCP method of such a two-mass system does not make it possible to build a three-loop SSR without taking into account the feedback of the elastic deformation moment $M_{12}(p)$, so we will not consider it.

The first option
An integral version of the synthesis by the generalized characteristic polynomial method

The TF $W_p(p)=\varphi_c(p)/U_s(p)$ for a positional three-loop SSR by the rescue cage rotation mechanism taking into account the elastic properties of the boom (Fig. 1) has the following form:

$$W_p(p) = \frac{W_{cp}(p)W_{cs2}(p)W_{cs1}(p)W_{el.dr}(p)\left(b_{12}p + C_{12}\right)}{J_1 p^2\left(J_2 p^2 + b_{12}p + C_{12}\right) + \left(b_{12}p + C_{12}\right)J_2 p^2 + \rightarrow}$$

$$\overline{\rightarrow + W_{cs1}(p)W_{el.dr}(p)K_{s1}p\left(J_2 p^2 + b_{12}p + C_{12}\right) + \rightarrow}$$

$$\overline{\rightarrow + W_{cs2}W_{cs1}(p)W_{el.dr}(p)\left(b_{12}p + C_{12}\right)K_{s2}p + \rightarrow}$$

$$\overline{\rightarrow + W_{cp}(p)W_{cs2}(p)W_{cs1}(p)W_{el.dr}(p)\left(b_{12}p + C_{12}\right)K_p}. \qquad (3)$$

Analysis (3) shows that the quality of the transition process for the initial coordinate $\varphi_c(p)$ will be affected by the poles and zeros of the system. The synthesis of the studied positional SSR by the rescue cage rotation mechanism will be carried out using the integral method of GCP [9].

As a result, we get the following system of equations:

$$\left.\begin{aligned} &\frac{\omega_0{}^3 J_1\left(J_2 p^2+b_{12} p+C_{12}\right)+J_2\left(b_{12} p+C_{12}\right)}{K_p W_{cp}(p) W_{cs2}(p) W_{cs1}(p) W_{el.dr}(p)\left(b_{12} p+C_{12}\right) p}=1,\\ &\frac{\omega_0{}^3 K_{s1}\left(J_2 p^2+b_{12} p+C_{12}\right)}{K_p W_{cp}(p) W_{cs2}(p)\left(b_{12} p+C_{12}\right) p}=\alpha_1 \omega_0,\\ &\frac{\omega_0{}^3 K_{s2}}{K_p W_{cp}(p)}=\alpha_2 \omega_0{}^2. \end{aligned}\right\} \quad (4)$$

where ω_0 is the geometric mean root of the standard form of the characteristic polynomial; α_1, α_2, α_3 are coefficients of the selected standard form of the characteristic polynomial.

After solving the obtained system of equations (4) taking into account (1), we obtain the following system of equations:

$$\left.\begin{aligned} &W_{cs1}(p)=\frac{\alpha_1 \omega_0 (T_\mu p+1)}{K_{s1} K_{el}}\left[J_1+\frac{J_2(b_{12} p+C_{12})}{J_2 p^2+b_{12} p+C_{12}}\right],\\ &W_{cs2}(p)=\frac{\alpha_2 \omega_0 K_{s1}}{\alpha_1 K_{s2}}\left(\frac{J_2 p}{b_{12} p+C_{12}}+\frac{1}{p}\right),\\ &W_{cp}(p)=\frac{\omega_0 K_{s2}}{\alpha_2 K_p} \end{aligned}\right\} \quad (5)$$

Based on the obtained expressions (5), it can be concluded that the TF of the first angular speed (motor) and second (cradle) regulators are difficult to implement and have a high order. This greatly complicates their further practical implementation.

The second variant of the combined variant of the synthesis by the generalized characteristic polynomial method

Let's consider the combined version of the synthesis of SSR by the mechanism of turning the cradle in the horizontal plane by the GCP method [9]. To do this, we first synthesize, using the integral method, a two-mass SSR with the speed of the second mass with contours for regulating the speeds of the first and second masses. The elastic torque regulating loop is not provided here, as in the previous case. Then we synthesize the external controller of the cradle position for the position loop, which includes the two-mass SSR synthesized earlier.

The expression of the TF $W_{s2}(p) = \omega_2(p)/U_{cp}(p)$ for the internal two-loop two-mass SSR has the form:

$$W_{s2}(p)=\frac{W_{cs2}(p) W_{cs1}(p) W_{el.dr}(p)\left(b_{12} p+C_{12}\right)}{\left[J_1 p\left(J_2 p^2+b_{12} p+C_{12}\right)+\left(b_{12} p+C_{12}\right) J_2 p\right]+\rightarrow}$$

$$\overline{\rightarrow+\left[W_{cs1}(p) W_{el.dr}(p) K_{s1}\left(J_2 p^2+b_{12} p+C_{12}\right)\right]+\rightarrow}$$

$$\overline{\rightarrow+W_{cs2} W_{cs1}(p) W_{el.dr}(p)\left(b_{12} p+C_{12}\right) K_{s2}}. \quad (6)$$

Let us represent the GCP of such SSR as a polynomial of the second order. To do this, multiply and divide the first and second components (in square brackets) of the characteristic polynomial TF (6) by p. After that, the GCP will be obtained by dividing the components of the characteristic polynomial term by term by the expression of the numerator.

Then, from the condition of equality of the GCP and the characteristic polynomial of some standard form, provided that the order of the synthesized system is $n = 2$, we write down the following system of equations:

$$\left.\begin{aligned} &\frac{\omega_{os}{}^2\left[J_1\left(J_2 p^2+b_{12} p+C_{12}\right)+J_2\left(b_{12} p+C_{12}\right)\right]}{K_{s2} W_{cs2}(p) W_{cs1}(p) W_{el.dr}(p)\left(b_{12} p+C_{12}\right) p}=1,\\ &\frac{\omega_{os}{}^2 K_{s1}\left(J_2 p^2+b_{12} p+C_{12}\right)}{K_{s2} W_{cs2}(p)\left(b_{12} p+C_{12}\right) p}=\alpha_1{}^s \omega_{os}, \end{aligned}\right\} \quad (7)$$

where $\alpha_1{}^s$ is a parameter of the second-order standard form selected for the speed loop of the second mass; ω_{os} is the geometric mean root of the selected standard form.

After solving the system of equations (7), we get:

$$\left.\begin{aligned} &W_{cs1}(p)=\frac{\omega_{os}\alpha_1{}^s (T_\mu p+1)}{K_{s1} K_{el}}\left(J_1+\frac{J_2\left(b_{12} p+C_{12}\right)}{\left(J_2 p^2+b_{12} p+C_{12}\right)}\right)\\ &W_{cs2}(p)=\frac{\omega_{os} K_{s1}}{\alpha_1{}^s K_{s2}}\left(\frac{J_2 p}{b_{12} p+C_{12}}+\frac{1}{p}\right). \end{aligned}\right\} \quad (8)$$

Now we synthesize the external loop of the position. To do this, based on the analysis of the structural diagram with an optimized internal SSR, which includes the speed controllers synthesized above, we will write the PF for the position loop $W_p(p) = \varphi_c'(p)/U_s(p)$:

$$W_p(p)=\frac{W_{cp}(p)\omega_{os}{}^2}{K_{s2} p^3+K_{s2}\alpha_1{}^s \omega_{os} p^2+K_{s2}\omega_{os}{}^2 p+W_{cp}(p) K_p \omega_{os}{}^2}. \quad (9)$$

Equating the expressions for the same powers of the GCP and the characteristic polynomial of the standard form under the condition $n = 3$, we obtain a system of equations, on the basis of which it is possible to calculate the expressions of the TF of the position controller $W_{cp}(p)$ and the value of the geometric mean root ω_{op} for the third-order system. Let's solve these equations under the condition that the parameters of the standard form of the third order are the values α_1^p, α_2^p.

$$\left.\begin{aligned} &W_{cp}(p)=\frac{{\omega_{op}}^3K_{s2}}{{\omega_{os}}^2K_p},\\ &\omega_{os}=\omega_{op}\frac{\alpha_1^p}{\alpha_1^s},\\ &\omega_{os}=\omega_{op}\sqrt{\alpha_2^p}. \end{aligned}\right\}. \qquad (10)$$

It is obvious that the last two conditions of expression (10) will be strictly fulfilled if $\sqrt{\alpha_2^p}=\alpha_1^p/\alpha_1^s$, i.e., the position loop is subject to requirements regarding the selection of a standard form for it. In the case when it is still chosen arbitrarily, it is worth choosing one for which $\sqrt{\alpha_2^p}$ and α_1^p/α_1^s it will not differ much. So, if you choose the standard forms of Butterworth, then $\alpha_1^s=\sqrt{2}$; α_1^p = α_2^p = 2. It is obvious that here the condition of ensuring this form in all loops is fulfilled. If we take the standard binomial form, when α_1^s = 2; α_1^p = α_2^p = 3, then we get contradictory conditions: ω_{os}= $3/2\,\omega_{op}$ and ω_{os} = $\sqrt{3}\,\omega_{op}$, which are not very different, and therefore a compromise is possible when adjusting all loops to the standard binomial form.

The third variant of the combined variant of the synthesis by the generalized characteristic polynomial method

Let's consider another combined version of the synthesis of SSR by the mechanism of turning the cradle by the GCP method [9]. To do this, we first synthesize the internal speed loop of the first mass, having brought the action of the internal connection by the torque of elastic deformations M_{12} to the input of the first speed controller CS1. Then we synthesize by the integral method a two-mass SSR with a cradle rotation mechanism taking into account the speed loop of the second mass and the synthesized speed loop of the first mass.

The expression of the TF $W_{s1}(p)=\omega_2(p)/U_{cp}(p)$ for the internal two-loop two-mass SSR has the form:

$$W_{s1}(p)=\frac{W_{cs1}(p)K_{el}}{J_1p\left(T_\mu p+1\right)+W_{cs1}(p)K_{el}K_{s1}}. \qquad (11)$$

Let us represent the GCP of such SSR as a polynomial of the first order. Then, from the condition of equality of the GCP and the characteristic polynomial of some standard form, provided that the order of the synthesized system is n = 1, we write down the following system of equations:

$$\left.\begin{aligned} &\frac{\omega_{os1}J_1\left(T_\mu p+1\right)}{K_{s1}W_{cs1}(p)K_{el}}=1,\\ &\frac{\omega_{os1}K_{s1}}{K_{s1}}=\omega_{os1}, \end{aligned}\right\}, \qquad (12)$$

where ω_{os1} is the geometric mean root of the selected standard form.

After solving the system of equations (12):

$$W_{cs1}(p)=\frac{\omega_{os1}J_1\left(T_\mu p+1\right)}{K_{el}K_{s1}} \qquad (13)$$

So, the speed controller of the first mass $W_{cs1}(p)$ turned out to be a PD controller.

We now synthesize by the integral method a two-mass SSR with a cradle rotation mechanism, taking into account the speed loop of the second mass and the synthesized loop of the speed control of the first mass.

To do this, based on the analysis of the structural diagram, which includes the speed controller of the first mass synthesized above, we will write the TF for the position loop $W_p(p)$ = $\varphi_c(p)$/U$_s$(p):

$$W_p(p)=\frac{W_{cp}(p)W_{cs2}(p)\dfrac{\omega_{os1}}{K_{s1}}J_1\left(b_{12}p+C_{12}\right)}{\left[J_1\left(p+\omega_{os1}\right)\left(J_2p^2+b_{12}p+C_{12}\right)+J_2p\left(b_{12}p+C_{12}\right)\right]p+\to}$$

$$\overline{\to+W_{cs2}(p)J_1\dfrac{\omega_{os1}}{K_{s1}}K_{s2}p\left(b_{12}p+C_{12}\right)+\to}$$

$$\overline{\to+W_{cs}(p)W_{cs2}(p)\dfrac{\omega_{os1}}{K_{s1}}J_1\left(b_{12}p+C_{12}\right)K_p}. \qquad (14)$$

Let us represent the GCP of such SSR as a polynomial of the second order. To do this, multiply and divide the first and second components (in square brackets) of the characteristic polynomial of the transfer function (14) by p. After that, the GCP will be obtained by dividing the components of the characteristic polynomial term by term by the expression of the numerator. Then, from the condition of equality of the GCP and the characteristic polynomial of some standard form, provided that the order of the synthesized system is $n = 2$, we write down the following system of equations:

$$\left.\begin{array}{l}\dfrac{\omega_{op}{}^2\left[J_1(p+\omega_{os1})(J_2p^2+b_{12}p+C_{12})+J_2(b_{12}p+C_{12})\right]}{K_pW_{cp}(p)W_{cs2}(p)\dfrac{\omega_{os1}}{K_{s1}}J_1(b_{12}p+C_{12})\,p}=1,\\[2ex] \dfrac{\omega_{op}{}^2K_{s2}}{K_pW_{cp}(p)}=\alpha_1{}^n\omega_{op},\end{array}\right\} \quad 15)$$

$\alpha_1{}^p$ is a parameter of the second-order standard form of the position selected for the loop; ω_{os} is the geometric mean root of the selected standard form.

After solving the system of equations (15):

$$\left.\begin{array}{l}W_{cs2}(p)=\dfrac{\omega_{op}(p+\omega_{os1})(J_2p^2+b_{12}p+C_{12})}{\dfrac{K_{s2}}{\alpha_1{}^p}\dfrac{\omega_{os1}}{K_{s1}}(b_{12}p+C_{12})p}+\dfrac{J_2\omega_{op}}{J_1\dfrac{K_{s2}}{\alpha_1{}^p}\dfrac{\omega_{os1}}{K_{s1}}}\\[2ex] W_{cp}(p)=\dfrac{\omega_{op}K_{s2}}{\alpha_1{}^pK_p}.\end{array}\right\}(16)$$

From the obtained results (13) and (16), it can be seen that the TF of the angular speed controllers of the motor and the cradle, which were obtained in the process of synthesis by the GCP method of the three-loop SSR by the cradle rotation mechanism, have a high order and turned out to be quite complex from the point of view of practical implementation. However, these systems make it possible to provide an aperiodic transition process of turning the cradle and low sensitivity, in a steady state, to the action of disturbances. Let's replace these controllers by approximation by evolutionary methods with more compact $PI^{\lambda}D^{\mu}$-controllers.

SYNTHESIS OF FRACTIONAL-ORDER CONTROLLERS FOR A TWO-MASS POSITION SSR WITH A CRADLE ROTATION MECHANISM

Having considered three variants of the synthesis of a two-mass positional SSR by the cradle rotation mechanism, it can be concluded that the TF of the angular speed of the motor and the cradle are complex and have a high order. This complicates their practical implementation. In addition, in the second option, contradictory conditions were obtained.

The use of the $PI^{\lambda}D^{\mu}$-controller of the FO is an effective approach to the optimization of various EMSs with the possibility of taking into account the effect of the two-mass effect of the control object based on the use of various intelligent methods

$$W_p(p)=K_p+\frac{1}{T_is^{\lambda}}+T_ds^{\mu}, \quad (17)$$

where K_p, T_i, T_d, are values of the proportional, integral and differential components.

Let's consider the first version of the synthesis as an example

With the other two options, the transition process is identical. Let's replace the complex regulators of angular speeds of the motor (CS1) $W_{cs1}(p)$ and cradle (CS2) $W_{cs2}(p)$ with TF (1) $PI^{\lambda}D^{\mu}$-controllers (17) DP by applying evolutionary methods for approximation [20].

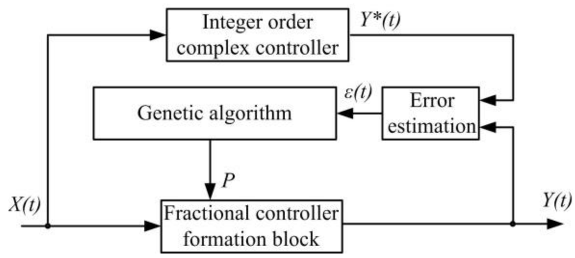

Fig. 2. **Functional scheme of approximation of high-order TFs by fractional TFs**
Source: **compiled by the authors**

The procedure for approximating a complex TF using one of the intelligent methods, in particular a genetic algorithm (GA) or a particle swarm, consists in comparing at each iteration of the transition process the initial integer TF of the complex controller *Y*(t)* and the transition function from the “approximating” TF *Y(t)*, which corresponds to the current setting of the parameters of the fractional controller. The difference between the instantaneous values of two TF controllers at certain discrete moments of time is analyzed by a special “error estimation” block. Through this block, the “intelligent method” of the GA or particles swarm corrects the parameters of the “fractional controller forming block” at each iteration until the minimum set error of divergence is reached. "Block of formation of the fractional controller" after achieving the desired accuracy will fix the parameters of the fractional controller, which approximates the transition function of the integer complex controller.

For the practical approximation of the classical high-order controller CS1 with PF $W_{cs1}(p)$ (5) by the $PI^{\lambda}D^{\mu}$-controller (17), a structural diagram was developed in the MATLAB package (Fig. 3).

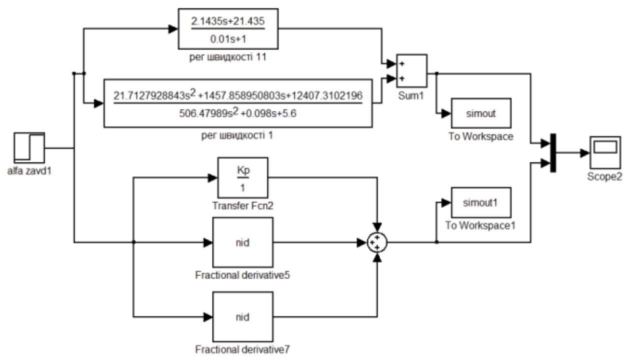

Fig. 3. **Structural diagram in MATLAB Simulink for practical approximation of the controller CS1 with TF (5) and (17)**
Source: **compiled by the authors**

The replacement of the high-order integer controller CS1 with the TF (5) was carried out in accordance with [20]. The GA method is implemented in MATLAB based on the Optimization Toolbox optimization package. The search for the parameters of the fractional $PI^{\lambda}D^{\mu}$-controller was carried out within the limits given in the Table. 1.

TABLE1. **LIMITS OF CHANGES OF $PI^{\Lambda}D^{M}$-CONTROLLERS PARAMETERS**

Parameter	Margin
K_p	[1..50]
$K_i = 1/T_i$	[0..100]
λ	[0..0.999]
T_d	[0..100]
μ	[0..0.999]

Source: **compiled by the authors**

As a result of the approximation, the TF of the $PI^{\lambda}D^{\mu}$-controller of the angular speed of the motor was obtained:

$$W_{cs1}'(p) = 12{,}197 + 12{,}241 p^{-0{,}185} + 2{,}434 p^{0{,}957}. \quad (18)$$

In Fig. 4 shows the transition functions for the RSh1 controller with the TF $W_{cs1}(p)$ (5) (curve 1) and the $PI^{\lambda}D^{\mu}$-controller with the TF $W_{cs1}'(p)$ (18) (curve 2) obtained by approximation.

In Fig. 5 shows the dependence of the accuracy on the time of the approximation process for finding the parameters of the $PI^{\lambda}D^{\mu}$-controller of the FO using GA. The population size is chosen to be 50. The approximation program shows the best (curve 1) and average values (curve 2) for the quality function J. By controlling these values, you can set the required accuracy to complete the approximation process. Additionally, we estimate the approximation result by the absolute root mean square deviation σ_n .

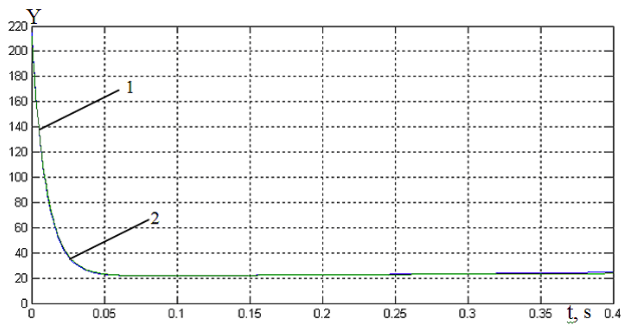

Fig. 4. **Transition functions of the motor angular speed controllers:
curve 1 – TF (5); curve 2 – TF (18)**
Source: **compiled by the authors**

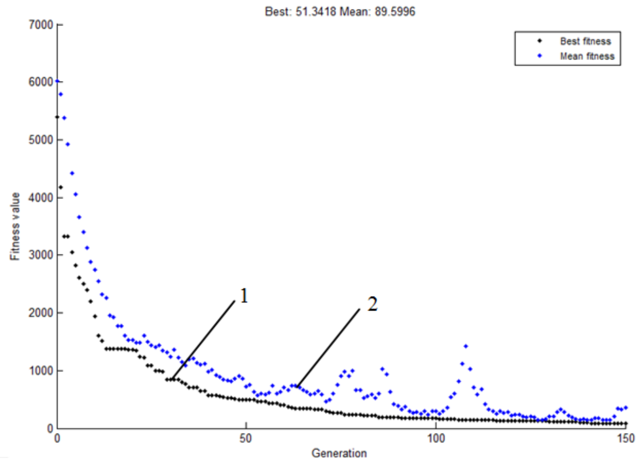

Fig. 5. **Dependence of the quality function on the iteration number for the approximation process using GA:
curve 1 – best; curve 2 – mean fitness**
Source: **compiled by the authors**

The research results practically confirm the thesis about the effectiveness of replacing the integer high-order controller CS1 with PF $W_{cs1}(p)$ (5) by a fractional $PI^{\lambda}D^{\mu}$-controller (17), whose TF is determined by approximating the transition function of the controller with TF (18) by the GA method.

At the second stage, we carry out a similar replacement of the high-order integer controller CS2 with the PF $W_{cs2}(p)$ (5) by the $PI^{\lambda}D^{\mu}$-controller (17) using the developed structural diagram in the MATLAB package (Fig. 6).

Similarly, we will use the Optimization Toolbox optimization package for approximation by the GA method and, taking the transition process of TF CS2 (1) as a reference, we will carry out the synthesis of the approximating $PI^{\lambda}D^{\mu}$-controller of the FO using the GA method.

As a result of the approximation, the following PF of the $PI^{\lambda}D^{\mu}$-controller of the angular speed of the cradle was obtained

$$W_{cs2}'(p) = 0{,}135 + 0{,}248 p^{-0{,}931} + 60{,}539 p^{0{,}978}. \quad (19)$$

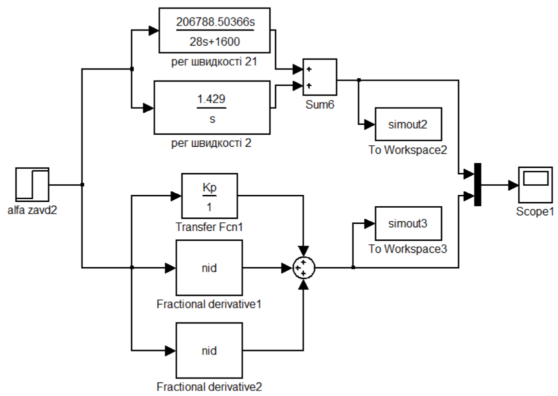

Fig. 6. **Structural diagram in MATLAB Simulink for practical approximation of the CS2 controller with TF (5) and (17)**
Source: **compiled by the authors**

In Fig. 7 shows the transition functions of the CS2 regulator with the TF $W_{cs2}(p)$ (5) (curve 1) and the controller with the TF $W_{cs2}(p)'$ (19) (curve 2) obtained using the GA method.

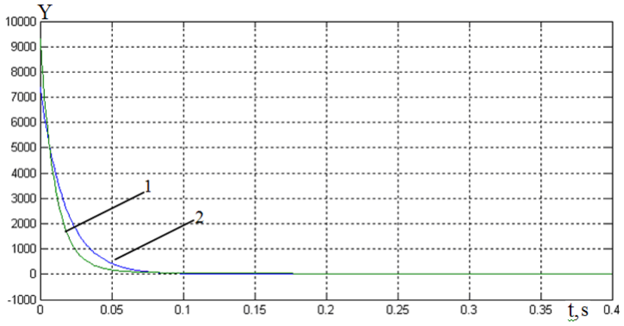

Fig. 7. **Transition functions of the cradle angular speed controllers:**
curve 1 – TF (5); curve 2 – TF (19)
Source: **compiled by the authors**

In Fig. 8 shows the dependence of the accuracy on the time of the approximation process for finding the parameters of the $PI^{\lambda}D^{\mu}$-controller of the FO using GA. The selected population size is 100.

The analysis of the obtained results of the approximation process of the complex controller CS2 with TF (5) and (19) found in the process of simulation modelling demonstrated satisfactory accuracy.

We conducted a comparative study of the system shown in Fig. 1 with integer-order controllers (5) and the same system with approximated fractional-order controllers (18) and (19). In Fig. 9 shows the resulting transition functions of the entire system (Fig. 1) with integer controllers (5) (curve 1) and of the system with $PI^{\lambda}D^{\mu}$-controller s (18) and (19) (curve 2) obtained using the GA method. At a time of 15 seconds, there is a step load jump, which both systems work well.

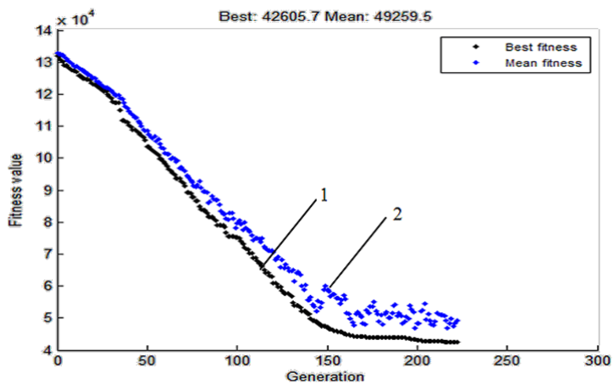

Fig. 8. **Dependence of the quality function on the iteration number for the approximation process using GA:**
curve 1 – best; curve 2 – mean fitness
Source: **compiled by the authors**

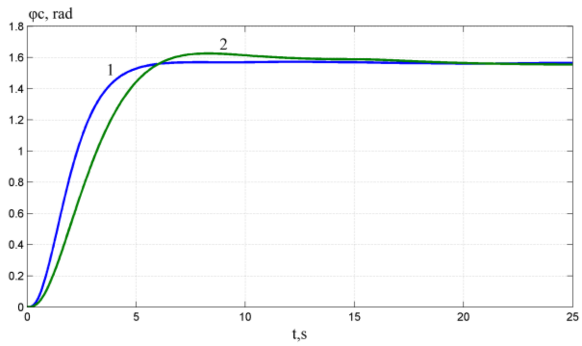

Fig. 9. **Transitional functions of the rescue cage position:**
curve 1 – TF (5); curve 2 – TFs (18) and (19)
Source: **compiled by the authors**

The analysis of the obtained results of the approximation process of complex controllers CS1 with TF (5) and (18), CS2 with TF (5) and (19) found in the process of simulation modelling demonstrated the satisfactory accuracy and workability of this approach. The accuracy of the approximation of the transition function of the controller of the angular speed of the rescue cage can be further increased by using not a compact $PI^{\lambda}D^{\mu}$-controller for CS2, but a slightly more complex fractional controller. Such a fractional controller does not complicate the implementation of ACS.

CONCLUSIONS

1. The article proposes a new and original approach to the synthesis of the loops of the automatic control systems of two-mass electromechanical systems based on the application of the generalized characteristic polynomial at the first stage and the intelligent optimization method at the second.

2. Using the generalized characteristic polynomial method, a positional three-loop system of subordinate control of the rescue cage rotation mechanism was created, taking into account the elastic properties of the boom with a rescue cage. The transition functions of the angular velocity controllers of the motor and rescue cage obtained in the process of synthesis are of a high order and turned out to be quite complex from the point of view of practical implementation.

3. Complex high-order controllers for the motor and angular speed of the rescue cage obtained as a result of the synthesis are replaced by more compact fractional-order $PI^{\lambda}D^{\mu}$-controllers by approximation using evolutionary methods. The conducted research with the help of mathematical modelling confirmed the effectiveness of replacing complex high-order angular speed controllers of the motor and high-order rescue cage with fractional-order $PI^{\lambda}D^{\mu}$-controllers, the transfer functions of which are determined by approximating the transition functions of the controllers using a genetic algorithm.

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Генетичний алгоритм переходу з високого на дробовий порядок регуляторів двомасової позиційної електромеханічної системи

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АНОТАЦІЯ

В статті запропоновано новий оригінальний підхід до синтезу контурів систем автоматичного керування двомасових позиційних електромеханічних систем на основі застосування узагальненого характеристичного поліному на першому етапі та інтелектуального методу оптимізації на другому. Підіймальний механізм пожежного автопідіймача на практиці є складним об’єктом керування. Недосконаліcть виготовлення механічних компонентів та їх з'єднань, пружні деформації стріли під час роботи та подачі вогнегасної речовини викликають коливання люльки. Використання системи автоматичного керування дає можливість демпфувати пружні коливання стріли. Синтезована система автоматичного керування, яка керує рухом стріли, повинна відповідати таким вимогам: необхідній швидкодії, статичній та динамічній точності переміщення люльки, відсутності значних перерегулювань у перехідних режимах тощо. Для виконання цих вимог проведено аналіз різних систем автоматичного керування та методів їх синтезу. В результаті аналізу за допомогою методу узагальненого характеристичного поліному створено двомасову позиційну триконтурну систему підпорядкованого керування механізмом повороту люльки з урахуванням пружних властивостей стріли із люлькою. Синтезована система підпорядкованого керування дозволяє демпфувати пружні коливання, забезпечуючи бажані перехідні процеси механізму повороту люльки та низьку чутливість у сталому режимі до дії збурень. Одержані в процесі синтезу передавальні функції регуляторів кутових швидкостей двигуна і люльки мають високий порядок і виявилися досить складними з точки зору практичної реалізації. Пропонується замінити ці регулятори більш компактними регуляторами дробового порядку. Проведені дослідження за допомогою математичного моделювання підтвердили ефективність заміни регуляторів високого порядку кутової швидкості двигуна та люльки на регулятори дробового порядку, передавальні функції яких визначаються шляхом апроксимації передавальних функцій регуляторів за допомогою генетичного алгоритму.

Keywords: пожежний автопідіймач; двомасова електромеханічна система; оригінальний синтез регуляторів; перехід до регуляторів дробового порядку; генетичний алгоритм

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