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Arbitrary random variables and Wiman's inequality for analytic functions in the unit disc

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Consider the class $\mathcal{A}$ of an analytic function f in the disc $\mathbb{D} := \{z\colon |z| < 1\}$ of the form

$$f(z) = \sum_{n=0}^{+\infty} a_n z^n. \tag{1}$$

Let $M_f(r) = \max\{|f(z)|\colon |z| = r\}$, $\mu_f(r) = \max\{|a_n|r^n\colon n \geq 0\}$, $r > 0$, be the maximum modulus and the maximal term of series (1), respectively.

We consider the random analytic functions of the form

$$f(z,\omega) = f(z,\omega_1,\omega_2) = \sum_{n=0}^{+\infty} R_n(\omega_1)\xi_n(\omega_2)a_n z^n, \tag{2}$$

where $a_n \in \mathbb{C}\colon \lim\limits_{n\to+\infty} \sqrt[n]{|a_n|} = 1$, $\big(R_n(\omega)\big)$ is the Rademacher sequence, $\big(\xi_n(\omega)\big)$ is a sequence of complex-valued random variables (denote by Δ_φ) such that there exists a constant $\beta > 0$ and a function $\varphi(N,\beta)\colon \mathbb{N}\times\mathbb{R}_+ \to [1;+\infty)$ non-decreasing by N and β such that

$$\Big(\mathbf{E}\Big(\max_{0\leq n\leq N} |\xi_n|^\beta\Big)\Big)^{1/\beta} \asymp \varphi(N,\beta), \quad N\to+\infty, \quad \alpha = \varlimsup_{N\to+\infty} \frac{\ln\varphi(N,\beta)}{\ln N} < +\infty, \tag{3}$$

$$(\exists\gamma > 0)(\exists n_0 \in \mathbb{N})\colon\ \sup\{\mathbf{E}|\xi_n|^{-\gamma}\colon\ n \geq n_0\} < +\infty. \tag{4}$$

Such class of random analytic functions denote by $\mathcal{A}(\varphi,\beta)$.

We will use the following notations

$$N(r) = \Big[\frac{1}{1-r}\ln\frac{\mu_f(r)}{1-r}\Big]^m, \quad m = \Big[\alpha + \frac{2}{\beta}\Big] + 4,$$

where $[x]$ means integer part of x.

We obtain the asymptotic estimates for maximum modulus of functions $f \in \mathcal{A}(\varphi,\beta)$. Here sequence $\big(\xi_n(\omega)\big)$ may not be sub-gaussian and may be dependent.

Theorem 1 ([1]). *Let $\delta > 0$. For $f \in \mathcal{A}(\varphi,\beta)$ there exist $r_0(\omega) > 0$, a set $E(\delta) \subset (0;1)$ of finite logarithmic measure (i.e. $\int_E (1-r)^{-1}dr < +\infty$) such that for all $r \in (r_0(\omega);1)\backslash E$ we have with probability $p \in (0;1)$*

$$M_f(r,\omega) \leq \frac{\mu_f(r)}{(1-p)^{1/\beta}}\varphi(N(r),\beta)\Big((1-r)^{-2}\cdot\ln\frac{\mu_f(r)\varphi(N(r),\beta)}{(1-p)(1-r)}\Big)^{1/4+\delta}.$$

Let Ξ_ρ be the class of random variables $\big\{\xi_k(\omega)\big\}$ such that there exist constants $\rho > 0$ and $C_1 > 0$ such that for every $n \in \mathbb{Z}_+$ and any $t \in [0,+\infty)$ we have

$$P\{\omega\colon |\xi_k(\omega)| \geq t\} \leq 2\exp\left(-\frac{t^\rho}{C_1}\right). \tag{5}$$

Remark that Ξ_2 is the class of sub-gaussian random variables and Ξ_1 is the class of sub-exponential random variables.

Corollary 1. *Let $\delta > 0$ and $\xi_k(\omega) \in \Xi_\rho$. Then for every $f \in \mathcal{H}(\varphi,\beta)$, there exist $r_0(\omega) > 0$ and a set $E(\delta) \subset (0,1)$ of finite logarithmic measure such that for all $r \in (r_0(\omega),1)\setminus E$ with probability $p \in (0,1)$ we get*

$$M_f(r,\omega) \leq \frac{\mu_f(r)}{1-p}\left(\frac{1}{(1-r)^2}\cdot\ln\frac{\mu_f(r)}{(1-p)(1-r)}\right)^{1/4+\delta}.$$

Similar statements for random entire function one can find in [2].

[1] A.O. Kuryliak, M.R. Kuryliak, O.M. Trusevych, *Arbitrary random variables and Wiman's inequality for analytic functions in the unit disc*, Mat. Stud. **61**, no.1, (2024), 39–45.

[2] A.O. Kuryliak, O.B. Skaskiv, A.I. Bandura, *Arbitrary random variables and WimanвЂ™s inequality for entire functions*, Axioms. **13(11)** (2024), 793.

On the stability of the maximum term of functional series in a system of functions

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Let us denote by L_+ the class of positive continuous on $\mathbb{R}_+ := [0, +\infty)$ functions $l(t)$ such that $l(t) \uparrow +\infty$ $(t \to +\infty)$, and by $\mathcal{W}$ we denote the class of functions $w \in L_+$ such that $\int_1^{+\infty} x^{-2}w(x)dx < +\infty$.

Let $\mathcal{S}(f, \Lambda)$ be the class of positive convergent for all $x \geq 0$ the functional series of the form

$$F(x) = \sum\nolimits_{k=0}^{+\infty} a_k f(x\lambda_k), \tag{6}$$

where $\Lambda = (\lambda_k)$ is a sequence of non-negative numbers such that $\lambda_k \neq \lambda_j$ for all $k \neq j$, $a_k \geq 0$ $(k \geq 0)$, and a positive increasing to $+\infty$ function f on $[0; +\infty)$ such that $f(0) = 1$ and $\ln f(x)$ is a convex function on the same interval. By $\mathcal{S}_*(f, \Lambda)$ we denote the class of formal series of form (6) such that $a_n f(x\lambda_n) \to 0$ $(n \to +\infty)$ for every $x \in \mathbb{R}_+$, i.e., for every $x \in \mathbb{R}_+$ there exists the maximal term

$$\mu(x, F) = \max\{|a_n| f(x\lambda_n) \colon n \geq 0\} < +\infty.$$

In the case $f(x) \equiv e^x$, we denote $\mathcal{D}_*(\Lambda) := \mathcal{S}_*(f, \Lambda)$.

For a function $w \in L$ let us denote

$$B_w(x) = \sum_{n=0}^{+\infty} a_n e^{w(\lambda_n)} f(x\lambda_n).$$

From Theorem 2 and Theorem 3 ([3]), proved for entire multiple Dirichlet series, it follows the following statement.

Theorem A ([3], Theorem 2). *Let $w \in L$, $B_w \in D_*(\lambda)$ and condition*

$$\int_0^{+\infty} t^{-2} \ln \nu_0(t) dt < +\infty \tag{7}$$

satisfies, where $\nu_0(t) = \int\limits_0^t e^{w(x)} dn(x)$, $n(x) = \sum_{\lambda_n \leq x} 1$. Then relation

$$\ln \mu(x, F) = (1 + o(1)) \ln \mu(x, B_w) \tag{8}$$

holds as $x \to +\infty$ outside some set $E \subset [0; +\infty)$, $\operatorname{meas} E < +\infty$.

Theorem A implies the following corollary.

Corollary 1. *Let $F \in \mathcal{D}_*(\Lambda)$. If there exists a function $w \in L$ such that $F_w \in \mathcal{D}_*(\Lambda)$, $\ln \nu \in \mathcal{W}$ (here $\nu(t) = \sum_{\lambda_n \leq t} e^{w(\lambda_n)}$) and*

$$e^{-w(\lambda_n)} \leq b_n \leq e^{w(\lambda_n)} \quad (n \geq k_1), \tag{9}$$

then there exists a set $E \subset \mathbb{R}_+$ of finite Lebesque measure such that

$$\ln \mu(x, F) = (1 + o(1)) \ln \mu(x, B_+) = (1 + o(1)) \ln \mu(x, B_-) \tag{10}$$

as $x \to +\infty$ $(x \notin E)$.