JOURNAL OF ENGINEERING SCIENCES

Volume 13, Issue 1 (2026)

Dziuba L. F., Pylypchuk M. I., Chmyr O. Y., Pavliuk R. V., Rubinskyy Y. V. (2026). Substantiation of the design parameters of the abrasive wheel for sharpening woodworking saws. Journal of Engineering Sciences (Ukraine), Vol. 13(1), pp. A1–A9. https://doi.org/10.21272/jes.2026.13(1).a1

p-ISSN 2312-2498
e-ISSN 2414-9381

Substantiation of the Design Parameters of the Abrasive Wheel for Sharpening Woodworking Saws

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Article info:
Submitted: July 23, 2025
Received in revised form: November 23, 2025
Accepted for publication: December 9, 2025
Available online: January 5, 2026

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Abstract. The sharpening of woodworking saws is a crucial operation in ensuring high-quality wood cutting. To avoid burns, burrs, and bending of the saw tooth during sharpening, it is necessary to provide an appropriate temperature regime in the contact area of the abrasive wheel and the cutting edge of the saw tooth. Abrasive wheels with an intermittent working surface enable the creation of a temperature in the sharpening zone that does not reach the point of thermal saturation. The article aims to substantiate the design parameters of an abrasive wheel with an intermittent working surface for sharpening woodworking saws based on the model of thermal interaction between the abrasive wheel and the saw tooth. To determine the time during which the surface of the cutting edge of the saw tooth is thermally saturated, a mathematical model of the thermal interaction between the abrasive wheel and the saw tooth during sharpening was developed based on solving the Cauchy problem for the differential equation of thermal conductivity using the Fourier transform. The dependence of the thermal saturation time on the longitudinal feed rate of the wheel was found. The lengths of the cutting segments of the abrasive wheel working surface, as well as the lengths of intermittent gaps (non-cutting segments), were determined taking into account the time of thermal saturation of the saw tooth.

Keywords: energy-efficient machining, thermal load control, sustainable machining, mathematical model, advanced mathematical methods.

1 Introduction

The quality of wood cutting depends significantly on the preparation of the woodworking tool for its use. The decisive operation that guarantees high-quality sawing of wood is the sharpening of the blades.

After analyzing the state of the issue regarding the sharpening of steel woodcutting saw teeth, it has been found that in practice, saw teeth are often sharpened in a way that forms burns, burrs, and bends in the tips of the teeth. This is because high temperatures (up to 800–900 °C) arise in the sharpening zone, resulting in uneven tooth pitches, and it is not always possible to select an abrasive wheel with the proper design parameters for certain sharpening conditions.

One way to improve a tool for sharpening saw blades is to create an abrasive wheel with an intermittent working surface [1]. The primary advantage of this tool is that it creates a non-continuous (non-uniform) temperature field during the sharpening process of saw blades for wood cutting. As a result, the temperature on the saw tooth surface is reduced, which leads to higher productivity and better sharpening quality. Such wheels are particularly effective when used for sharpening frame and circular steel saws for cutting wood. Despite their significant advantages, abrasive tools with an intermittent working surface are not used for sharpening woodcutting tools, and the sharpening processes using such wheels have not yet been studied.

Therefore, it is essential to theoretically substantiate and experimentally study the effectiveness of using an abrasive wheel with an intermittent working surface for sharpening woodcutting saws, which will increase the wear resistance of saw teeth and improve the quality of the sawing process.

2 Literature Review

High temperatures are generated when sharpening saw teeth with abrasive wheels, as well as when grinding workpiece surfaces with an abrasive tool. As noted in [2], a technical issue with grinding is the potential for a temperature increase that can lead to thermal damage to the surface. It is crucial to ensure the surface integrity of the workpiece during the grinding process. This requires the development of mathematical models of the temperature field that account for various parameters, including workpiece materials, grinding wheel types, grinding mode parameters, and cooling methods. This research thoroughly analyzes and summarizes the temperature field models during the grinding process. An analytical model for estimating the temperature of cooled grinding is presented in [3]. The maximum increase in surface grinding temperature is calculated for the cooled grinding process, taking into account the total heat flux entering the system. It was found that at temperatures above 631°C, burning from grinding is observed on the working surface of microalloyed steel.

The research [4] states that heat generation is a critical problem during grinding. If grinding generates significant heat, the dimensional and shape accuracy of the workpiece can be compromised due to thermal deformation, and the machined surface can be degraded by burning from excessive heat.

Avoiding burning due to high temperatures is particularly important when finishing tooth wheels. The process of tooth wheel grinding is examined in [5], where the effect of coolant on temperature during continuous grinding with an abrasive tool in the form of a worm cutter is studied. The area of contact between the abrasive tool and the tooth wheel is considered a moving heat source. The article evaluates the efficiency of cooling by the liquid supplied to the grinding zone.

The effect of coolant on the grinding process is also discussed in [6]. In this work, a thermal model of spatial interaction between a small-module gear and an abrasive wheel was developed, and the change in thermal load due to variations in coolant supply conditions during grinding with a solid wheel was experimentally investigated. The modeling results were confirmed experimentally with an accuracy of up to 10 %.

Abrasive wheels with an intermittent working surface are widely used for grinding gear teeth and cylindrical surfaces. The research [7] examines the influence of the thermal factor during surface (flat) abrasive grinding and confirms the hypothesis that abrasive wheels with intermittent working surfaces can reduce the temperature in the cutting zone, similar to highly porous wheels.

In [8], it is stated that during the grinding process, temperatures of 800–1600 °C occur in the contact zone between the abrasive tool and the workpiece being processed. Such high temperatures cause phase transformations in the surface layers of the workpiece material, resulting in the appearance of residual tensile stresses.

To reduce the thermal stress on the surface layer of a workpiece, research [10] suggests using abrasive wheels with an intermittent working surface. This study examines the impact of the geometric parameters of abrasive wheels with intermittent working surfaces on the relative wear of surfaces, the temperature in the cutting zone, and the parametric stability of elastic systems in gear-tooth grinding and surface grinding machines. A comparative analysis of abrasive wheels with continuous and intermittent working surfaces for finishing abrasive machining of roller bearing rings was conducted in [9], which demonstrated that the intermittent working surface of the abrasive tool enables machining within the permissible contact temperature range. The research [10] states that, compared to grinding with solid wheels, intermittent wheels have a higher cutting capacity when operating in self-sharpening mode. Additionally, it is indicated that with a decrease in the length of the cutting segment of the abrasive wheel, the amplitude of its oscillations decreases.

The research [11] investigates the wear intensity of abrasive wheels with an intermittent working surface. The study [12] states that grinding is characterized by high thermal stress, which negatively affects the machining of the workpiece. When residual tensile stresses are generated in the surface layer of the material, microhardness decreases, and burn-in and microcracks appear. To avoid this, it is essential to understand the actual causes of temperature rise and the appearance of temperature defects in machining, as well as to have analytical solutions regarding the patterns of occurrence of grinding temperature [2].

One of the theoretical approaches to determining the average grinding temperature involves considering the grinding zone as a “contact spot” between the wheel and the material being processed, within which the heat source operates [2, 12]. Therefore, there is a need to develop mathematical models of the thermal interaction between the abrasive wheel and the workpiece being processed.

A mathematical model for determining the temperature of intermittent dry grinding and using forced cooling at the intervals of thermal macro- and microcycles was developed and studied in [13]. In this work, the temperature field of intermittent dry grinding is formed by superimposing temperature fields at the stages of heating and cooling cycles, resulting from the influence of heat flux at each point on the surface being ground.

A comparative analysis of the operation of abrasive wheels with intermittent and solid surfaces was performed in [14]. The authors recommend using intermittent abrasive wheels for operations without center grinding of roller bearing rings. The research presents a comparative analysis of the use of abrasive wheels with a diameter of

300 mm, featuring both continuous and intermittent working surfaces, for grinding hardened steel Kh12F1. An abrasive wheel with ten gaps, each 47 mm in length, which was equal to the length of the cutting segments, allowed for a 30 % reduction in temperature in the grinding zone. The authors of [14] conclude that the reduction of thermal stress during grinding with intermittent wheels enables the intensification of processing modes without the risk of burning on the processed surfaces.

Grinding the cutting edges of a tool is a crucial step in the tool sharpening process. The influence of the grinding process parameters on the formation of burrs on the cutting edge of a carbide microdrill was experimentally studied [15]. The following grinding parameters are analyzed in this research: feed rate, depth of grinding, wheel service life, and wheel rotation speed. The characteristics of damage to the edge of a cutting blade made of WC-Co hard-alloy material, after precision grinding with a diamond wheel, were experimentally studied in [16]. The authors state that the angular speed of rotation of a solid abrasive wheel is one of the factors that influence the formation of damage during grinding in the form of tiny cracks and edge breakage.

The process of sharpening steel saws for cutting wood, as well as grinding workpieces, is also accompanied by the generation of heat. Heat is generated during the cutting of metal by the grain of the abrasive wheel, affecting the change in the properties of the surface layer of the sawtooth material. The heating of thin surface layers of metal during sharpening was studied experimentally in [1]. It was found that when sharpening the teeth of frame saws with a solid wheel, an increase in the feed rate of the wheel and cutting speed leads to a rise in temperature up to 728 °C over a specific time.

This work demonstrates that the use of an intermittent abrasive wheel (Figure 1) results in a 1.23-fold decrease in temperature on the saw tooth [1].

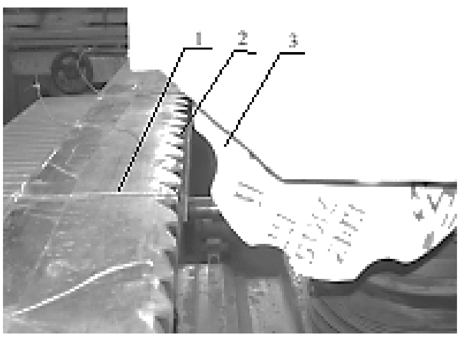

Figure 1 – Positioning the abrasive wheel in relation to the saw tooth [1]: 1 – band woodcutting saw; 2 – saw teeth; 3 – abrasive wheel with intermittent working surface

In this regard, it is essential to establish the dependence of the temperature in the area of contact between the wheel and the saw tooth surface on the number of wheel gaps and sharpening modes.

The research aims to substantiate the design parameters of an abrasive wheel with an intermittent working surface for sharpening woodcutting saws based on the model of thermal interaction between the abrasive wheel and the saw tooth.

To achieve the goal, the following tasks need to be solved:

– to develop a mathematical model of the thermal interaction between an abrasive wheel and a saw tooth during sharpening based on solving the Cauchy problem for the differential thermal conductivity equation using the Fourier transform;

– based on the developed mathematical model, to investigate the change in the time of thermal saturation of the saw tooth depending on the feed rate of the abrasive wheel and the material of the woodworking saw;

– to determine the lengths of the cutting segments of the working surface of the abrasive wheel with gaps and calculate the number of cutting segments, taking into account the time of thermal saturation of the saw tooth.

3 Research Methodology

3.1 A mathematical model of thermal interaction between an abrasive wheel and a saw tooth during sharpening

According to [2, 12], based on theoretical studies of various types of grinding, we believe that during the sharpening of the saw blade edge, the limiting state of the temperature field does not occur immediately after the beginning of sharpening. From the start of the sharpening to the onset of the limiting state, i.e., the state of thermal saturation, there is a physically small period of time during which the sharpening proceeds in a non-steady mode. The existence of such a time interval was confirmed experimentally in [1]. The presence of such a non-steady mode allows for reducing the temperature in the contact zone between the abrasive wheel and the saw's cutting edge by periodically interrupting the sharpening process.

During sharpening, the abrasive wheel moves continuously along the cutting edge of the tooth at a constant feed rate. The area of contact between the abrasive wheel and the cutting edge of the saw tooth can be considered a narrow rectangular strip that moves along the cutting edge of the saw tooth. In this case, the temperature field can be modeled using the scheme of an infinitely long, strip-like heat source in motion. Thus, a heat source can be considered to consist of an infinitely large number of simultaneously moving linear sources.

It is known that during sharpening without cooling, the heat released into the air near the contact zone of the abrasive wheel with the tooth is low. Therefore, the effect of heat release into the air can be neglected, and the sharpening surface can be considered isolated. Under this assumption, there is no heat transmission in the sharpening zone. It is assumed the heat source to be a strip of width $2h$ (Figure 2), moving at a constant speed v, which is equal to the longitudinal feed rate of the wheel v_s, m/min, along the axis z that coincides with the cutting edge of the saw tooth.

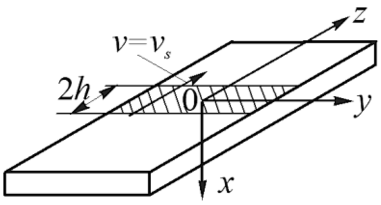

Figure 2 – A design scheme for the study of the thermal process during the sharpening of a saw tooth: x, y, z – coordinate axes; $v = v_s$ – speed of heat source movement, m/s; $2h$ – width of heat source strip, m

The coordinate x is vertical, directed along the thickness of the saw tooth, and the coordinate y is directed perpendicular to the cutting edge of the tooth.

The change in temperature on the surface of the saw tooth will be investigated based on solving the classical differential thermal conductivity equation:

$$\frac{\partial\theta}{\partial\tau} = a\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}\right);$$
$$(x, y, z, \tau) \in R \times R \times R \times (0, T); \quad (1)$$
$$X = (x, y, z) \in R^3,$$

where θ – temperature on the surface of the cutting edge of the tooth, °C, which is the function of $\theta = \theta(X, \tau)$; τ – time of thermal saturation of the surface layer of the tooth material, c, time variation range from 0 to T; a – thermal conductivity of the material, m²/s; R – a set of real numbers.

At the moment when $\tau = 0$, the following initial condition can be added to the differential equation (1):

$$\theta(x, y, z, 0) = \frac{2Q}{c\gamma}, \quad (2)$$

where Q – the amount of heat, J, c – is the mass heat capacity of the saw tooth material, J/(kg·K); γ – the density of the saw tooth material, kg/m³.

The differential equation (1) and the initial condition (2) are a Cauchy problem, the solution of which will be found using the Fourier transform. As is known, for an arbitrary function $f(X)$, its Fourier transform $F(\xi)$ is determined by the formula:

$$\theta(x, y, z, 0) = \frac{2Q}{c\gamma}, \quad (3)$$

where $(X, \xi) = x\xi_1 + y\xi_2 + z\xi_3$.

The formula finds the original function by its Fourier transform:

$$f(X) = \frac{1}{(2\pi)^3}\int_{R^3} F(\xi)e^{i(X,\xi)}\,d\xi, \quad (4)$$

After denoting by $\theta(\xi, \tau)$ the Fourier transform of the unknown function $\theta(X, \tau)$ as

$$\theta(\xi, \tau) = \int_{R^3} \theta(X, \tau)e^{-i(X,\xi)}\,dX,$$

let's apply the Fourier transform to equation (1).

As a result, on the left-hand side, it can be obtained:

$$\int_{R^3} \theta_\tau(X, \tau)e^{-i(X,\xi)}\,dX = \theta_\tau(\xi, \tau). \quad (5)$$

To obtain the Fourier transform of the right-hand side of equation (1), it should first be perform integration by parts in the following integral:

$$\int_{-\infty}^{+\infty} \theta_{xx}(X, \tau)e^{-ix\xi_1}dx = \theta_x(X, \tau)e^{-ix\xi_1}\Big|_{x\to-\infty}^{x\to+\infty} +$$
$$+i\xi_1\int_{-\infty}^{+\infty} \theta_x(X, \tau)e^{-ix\xi_1}dx = \quad (6)$$
$$= -{\xi_1}^2\int_{-\infty}^{+\infty} \theta(X, \tau)e^{-ix\xi_1}dx,$$

where $\theta_x(X, \tau)e^{-ix\xi_1}\Big|_{x\to-\infty}^{x\to+\infty} = 0$ by the necessary condition of convergence on $\pm\infty$ ($i^2 = -1$).

Similarly to (6), it can be obtained:

$$\int_{-\infty}^{+\infty} \theta_{yy}(X, \tau)e^{-iy\xi_2}dx = -{\xi_2}^2\int_{-\infty}^{+\infty} \theta(X, \tau)e^{-iy\xi_2}dx;$$
$$\int_{-\infty}^{+\infty} \theta_{zz}(X, \tau)e^{-iz\xi_3}dx = -{\xi_3}^2\int_{-\infty}^{+\infty} \theta(X, \tau)e^{-iz\xi_3}dx.$$

Taking into account (6) and the above dependencies, it can be obtained:

$$\int_{R^3} \Delta\theta(X, \tau)e^{-i(X,\xi)}\,dX =$$
$$= -\sum_{k=1}^{3}\xi_k^2\int_{R^3} \theta(X, \tau)e^{-i(X,\xi)}\,dX = \quad (7)$$
$$= -|\xi|^2\theta(\xi, \tau),$$

where $|\xi|^2 = \sum_{k=1}^{3}\xi_k^2$.

Thus, by applying the Fourier transform to the differential equation (1), after taking into account formulas (5) and (7) for the equation regarding the representation of the unknown function, it can be obtained:

$$\theta_\tau(\xi, \tau) + a|\xi|^2\theta(\xi, \tau) = 0. \quad (8)$$

After performing the Fourier transform of the initial condition (2) and introducing the notation:

$$\Phi(\xi) = \int_{R^3} \frac{2Q}{c\gamma}e^{-i(Y,\xi)}\,dY,$$

it can also be obtained:

$$\theta(\xi, 0) = \Phi(\xi). \quad (9)$$

Then, in the Fourier transform, it is necessary to solve equation (8) under the initial condition (9).

After integrating equation (8) and the transformations, we obtain $\theta(\xi, \tau) = C(\xi)e^{-a|\xi|^2\tau}$. Taking into account the initial condition (9) $\theta(\xi, 0) = C(\xi) = \Phi(\xi)$, it can be obtained:

$$\theta(\xi, \tau) = \Phi(\xi)e^{-a|\xi|^2\tau}. \quad (10)$$

The dependence (10) is the Fourier transform of the solution of the differential equation (1) under the initial condition (2).

To find the solution to (10), the inverse Fourier transform formula can be applied:

$$\theta(X, \tau) = \frac{1}{(2\pi)^3}\int_{R^3} \Phi(\xi)e^{-a|\xi|^2\tau + i(X,\xi)}\,d\xi. \quad (11)$$

Let's transform dependence (11) by replacing the representation $\Phi(\xi)$ with an explicit form according to (3):

$$\theta(X, \tau) = \frac{1}{(2\pi)^3}\int_{R^3} \frac{2Q}{c\gamma}dY\int_{R^3} e^{-a|\xi|^2\tau + i(X-Y,\xi)}\,d\xi. \quad (12)$$

After applying Euler's formula to (12), it can be obtained:

$$\int_{R^3} e^{-a|\xi|^2\tau+i(X-Y,\xi)}\,d\xi = \\ = \prod_{k=1}^{3}\int_{-\infty}^{+\infty} e^{-a\xi_k^2\tau}\,[(cos(x_k-y_k)\xi_k + \\ +i\,sin(x_k-y_k)\xi_k]d\xi_k = \\ = \prod_{k=1}^{3}\int_{-\infty}^{+\infty} e^{-a\xi_k^2\tau}\,cos(x_k-y_k)\xi_k\,d\xi_k. \tag{13}$$

In the dependence (13), it is assumed that $x_1 = x$, $x_2 = y, x_3 = z, y_1 = x', y_2 = y', y_3 = z'$, where x', y', z' are the coordinates of the heat source at time τ.

Let's enter the notation:

$$I(s) = \int_0^{+\infty} e^{-\alpha x^2}\,cos\,s\,xdx, \tag{14}$$

where $s = x_k - y_k$ – parameter, α – number.

Determining the derivative $I'(s)$ from (14) and integrating by parts, the following equation can be obtained:

$$I'(s) + \frac{s}{2\alpha}I(s) = 0,$$

where $\alpha = a\tau$.

The solution to this equation is as follows:

$$I(s) = Ce^{-\frac{s^2}{4\alpha}},$$

where the integration constant C is defined under the condition for $s = 0$:

$$C = I(0) = \int_0^{+\infty} e^{-\alpha x^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}.$$

Taking into account (14) and the integration constant, it can be obtained:

$$\int_0^{+\infty} e^{-\alpha x^2}\,cos\,s\,xdx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}e^{-\frac{s^2}{4\alpha}}. \tag{15}$$

Taking into account dependencies (13) and (15), the dependency (12) can be reduced to the Poisson formula:

$$\theta(X,\tau) = \frac{1}{(2\sqrt{\pi a\tau})^3}\int_{R^3}\frac{2Q}{c\gamma}e^{-\frac{(X-Y)^2}{4a\tau}}dY. \tag{16}$$

If the heat source moves along the saw tooth in the positive direction of the axis z (Figure 2), the coordinate z' changes continuously by the value of $v\tau$, where v – the speed of the heat source, which is the abrasive wheel.

When the abrasive wheel moved for a time period τ up to the considered time, then we solve the differential equation (1) under the initial condition (2) by integrating equation (16) along y' from $-\infty$ to $+\infty$, and along z' – from 0 to $2h$, where h – half the width of the band heat source (Figure 2) over time from 0 to τ, i.e.:

$$\theta(X,\tau) = \int_0^{2h} dz'\int_{-\infty}^{+\infty} dy'\int_0^{\tau}\frac{2q}{c\gamma}\frac{1}{\sqrt{(4\pi a\tau)^3}}e^{-\frac{x^2+(y-y')^2+(z-z'+v\tau)^2}{4a\tau}}d\tau, \tag{17}$$

where q – heat flux density, i.e., the amount of heat that passes through a unit surface area of the cutting edge of the tooth per unit time.

The dependence (17) describes the temperature field created by the band heat source.

Next, let's consider the following integral:

$$\int_{-\infty}^{+\infty} e^{-\frac{x^2+(y-y')^2+(z-z'+v\tau)^2}{4a\tau}}dy' = \\ = e^{-\frac{x^2+(z-z'+v\tau)^2}{4a\tau}}\int_{-\infty}^{+\infty} e^{-\frac{(y-y')^2}{4a\tau}}dy', \tag{18}$$

in which a substitution $t = \frac{y-y'}{\sqrt{4a\tau}}$ can be made.

Then $y' = y - \sqrt{4a\tau}\cdot t$, $dy' = -\sqrt{4a\tau}\cdot dt$, and the expression (18) takes the following form:

$$e^{-\frac{x^2+(z-z'+v\tau)^2}{4a\tau}}\int_{+\infty}^{-\infty} e^{-t^2}\left(-\sqrt{4a\tau}\right)dt = \\ = \sqrt{4\pi a\tau}\cdot e^{-\frac{x^2+(z-z'+v\tau)^2}{4a\tau}}, \tag{19}$$

where $\int_{-\infty}^{+\infty} e^{-t^2}dt = \sqrt{\pi}$.

Let's substitute the dependence (19) into the expression (17):

$$\theta(X,\tau) = \int_0^{2h} dz'\int_0^{\tau}\frac{2q}{c\gamma}\frac{1}{\sqrt{(4\pi a\tau)^3}}\sqrt{4\pi a\tau}\cdot e^{-\frac{x^2+(z-z'+v\tau)^2}{4a\tau}}d\tau = \\ = \frac{q}{2\pi\lambda}\int_0^{2h} dz'\int_0^{\tau}\frac{1}{\tau}\cdot e^{-\frac{x^2+(z-z'+v\tau)^2}{4a\tau}}d\tau. \tag{20}$$

where the notation $\lambda = ac\gamma$ is introduced.

Let's consider the integral $\int_0^{2h} e^{-\frac{x^2+(z-z'+v\tau)^2}{4a\tau}}dz'$, in which a substitutions $z - z' = w$, $z' = z - w$ and $dz' = -dw$ can be made. At $z' = 0$: $w = z$; at $z' = 2h$: $w = z - 2h$; then:

$$\int_0^{2h} e^{-\frac{x^2+(z-z'+v\tau)^2}{4a\tau}}dz' = -\int_z^{z-2h} e^{-\frac{x^2+(w+v\tau)^2}{4a\tau}}dw.$$

By taking $x = 0$, which corresponds to the surface of the cutting edge of the tooth during sharpening with an abrasive wheel, the considered integral takes the form:

$$\int_0^{2h} e^{-\frac{x^2+(z-z'+v\tau)^2}{4a\tau}}dz' = -\int_z^{z-2h} e^{-\frac{(w+v\tau)^2}{4a\tau}}dw. \tag{21}$$

After a substitution $\frac{w+v\tau}{2\sqrt{a\tau}} = u$, then $w = 2\sqrt{a\tau}\cdot u - v\tau$, and $dw = 2\sqrt{a\tau}du$. At $w = z$: $u = \frac{z+v\tau}{2\sqrt{a\tau}} = u_1$, at $w = z - 2h$: $u = \frac{z-2h+v\tau}{2\sqrt{a\tau}} = u_2$.

With this substitution, the integral (21) takes the form:

$$-2\sqrt{a\tau}\int_{u_1}^{u_2} e^{-u^2}du = \\ = 2\sqrt{a\tau}\left(\int_0^{u_1} e^{-u^2}du - \int_0^{u_2} e^{-u^2}du\right) \\ \sqrt{\pi a\tau}[\Phi(u_1) - \Phi(u_2)], \tag{22}$$

where $\Phi(u)$ – the Gaussian probability integral, in which the arguments u_1, u_2 are always positive and differ from each other by the following value:

$$u_1 - u_2 = \frac{h}{\sqrt{a\tau}}. \tag{23}$$

To find out the value of the difference in the arguments of the Gaussian integral, let us analyze the process of sharpening the cutting edge of the saw. During sharpening, the width $2h$ (Figure 2) of the heat source band, which is the end of the abrasive wheel, usually does not exceed 2–3 mm.

Let's assume that at a low longitudinal feed rate of the abrasive wheel (up to 2 m/min), the thermal saturation time is 0.01 s, and the thermal conductivity coefficient of the tool steel is $a = 4\cdot10^{-6}$ m^2/s. Then the expression (23) acquires the following value:

$$\frac{h}{\sqrt{a\tau}} = \frac{1\cdot10^{-3}}{\sqrt{4\cdot10^{-6}\cdot0.01}} = 5.$$

Thus, at the initial time point before thermal saturation, the value of the argument u of the Gaussian probability integral $\Phi(u)$ is 5. With such argument values, the value $\Phi(u)$ is 1. In this case, the expression (22) can be represented as $\sqrt{\pi a\tau}[1-\Phi(u_2)]$, and the temperature field in the process of thermal saturation can be described by the following dependence:

$$\theta = \frac{q}{2\pi\lambda}\int_0^{\tau}\frac{\sqrt{\pi a\tau}}{\tau}[1-\Phi(u_2)]\ d\tau. \quad (24)$$

Next, we will perform the substitution $u_2 = \frac{v\sqrt{\tau}}{2\sqrt{a}}$ at $z = 2h$. Then $\tau = \frac{4au_2^2}{v^2}$, $d\tau = \frac{4\sqrt{a\cdot\tau}}{v}du_2$. At $\tau = 0$: $u_2 = 0$; also at $\tau > 0$, $u_2 = \frac{v\sqrt{\tau}}{2\sqrt{a}}$..

After taking into account these transformations in the expression (24), it can be obtained:

$$\theta = \frac{2aq}{\lambda v\sqrt{\pi}}\int_0^{\frac{v\sqrt{\tau}}{2\sqrt{a}}}[1-\Phi(u_2)]du_2. \quad (25)$$

After integrating (25), the following dependence of the temperature on the surface of the cutting edge on the sharpening time and the heat source speed can be obtained:

$$\theta = \frac{2aq}{\lambda v\sqrt{\pi}}\left\{\frac{v}{2}\sqrt{\frac{\tau}{a}}\left[1-\Phi\left(\frac{v}{2}\sqrt{\frac{\tau}{a}}\right)\right]-\frac{1}{\sqrt{\pi}}\left(e^{-\frac{v^2\tau}{4a}}-1\right)\right\}. \quad (26)$$

3.2 The change in the time of thermal saturation of the saw tooth, depending on the feed rate of the abrasive wheel

According to formula (26), the temperature θ will reach its highest value when the time τ is equal to the time of thermal saturation τ^*.

Taking into account that $\frac{v}{2}\sqrt{\frac{\tau^*}{a}}\left[1-\Phi\left(\frac{v}{2}\sqrt{\frac{\tau^*}{a}}\right)\right] = 0$ under the condition $\Phi\left(\frac{v}{2}\sqrt{\frac{\tau^*}{a}}\right) = 1$ at high values of the argument, we obtain from (26) the dependence of the thermal saturation time on the longitudinal feed rate of the wheel:

$$\tau^* = \frac{4a}{\pi v^2}ln^2(1-\theta^*), \quad (27)$$

where $\theta^* = \frac{\lambda\pi v}{2qa}\theta$ $(0 < \theta^* < 1)$.

3.3 Determination of the lengths for the cutting segments of the working surface in the intermittent abrasive wheel

Based on the value of the thermal saturation time, as specified in equation (27), the length l_1 of the cutting segment on the working surface of the abrasive wheel can be determined (Figure 3).

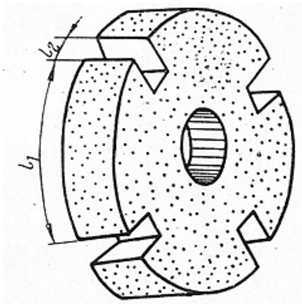

Figure 3 – Sketch of an abrasive wheel with an intermittent working surface: l_1 – length of the wheel's working surface cutting segments; l_2 – length of the wheel's intermittent working zone gaps

If the contact time between the area on the surface of the cutting edge of the saw and the surface of the abrasive wheel is shorter than the thermal saturation time τ^*, the temperature on the cutting edge surface does not reach its highest value. This can be achieved when a portion of the surface of the saw's cutting edge, after contacting the cutting segment on the wheel, falls into the gap zone (pores) of the wheel's intermittent working zone.

By changing the lengths of the cutting segments and gaps, it is possible to create conditions under which the sharpening temperature will only reach a predetermined value. The following formula determines the length of the cutting segments of the abrasive wheel:

$$l_1 = \frac{\pi Dn}{60}\tau^*, \quad (28)$$

where n – wheel rotation speed, rpm; D – diameter of the wheel around the cutting segment, mm.

To calculate the number of cutting segments, we assume that the lengths of the segments are equal to the lengths of the gaps.

Then the number of cutting segments is determined by the ratio of the circumference of the segments of the intermittent working surface of the wheel to the doubled height of the cutting segment:

$$N = \frac{\pi D}{2l_1}, \quad (29)$$

where D and l_1 have the same dimensions, mm.

4 Results

For the production of woodworking saws, alloyed medium carbon steels of grades 75Cr and D6A, as well as carbon steel of grade C75, are used. The 75Cr steel is alloyed with chromium, while the D6A steel contains alloying elements of Ni, Cr, Mo, and V.

For these steels, the thermal conductivity coefficient ranges from $a = 4\cdot10^{-6}$ m^2/s to $a = 8\cdot10^{-6}$ m^2/s.

Figure 4 shows the dependencies of the time of thermal saturation of saw teeth made of alloyed and carbon steel on the speed v, m/s of longitudinal feed of the abrasive wheel at the corresponding values of relative temperature θ^*.

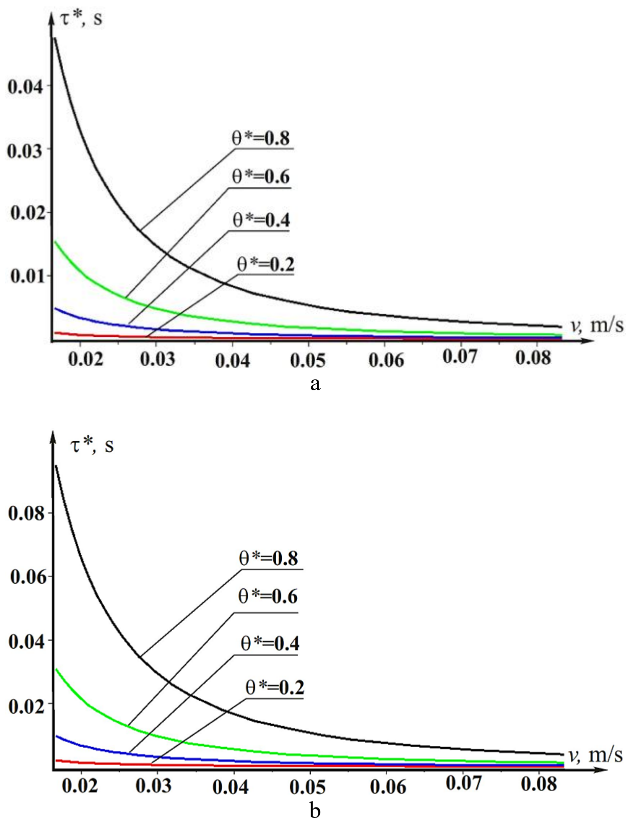

Figure 4 – Dependencies of the thermal saturation time on the linear speed of the abrasive wheel at different relative temperature values θ^*: a – $a = 4\cdot10^{-6}$ m^2/s; b – $a = 8\cdot10^{-6}$ m^2/s

The time τ^* reaches its highest value at low feed rates when $v = 0.03$ m/s.

When sharpening saw blades made of steels with a thermal conductivity coefficient of $a = 4\cdot10^{-6}$ m^2/s (Figure 4a) for a relative temperature $\theta^* = 0.2$, which corresponds to reaching a sharpening temperature of only 20 % of the thermal saturation temperature, the time τ^* is the shortest and does not depend on the wheel's feed rate.

At $\theta^* = 0.4$, the thermal saturation time from a value of 0.05 s at low feed rates moves to even smaller values with increasing speed v.

At $\theta^* = 0.6$, the value of time τ^* decreases from 0.017 s at low speed and asymptotically approaches zero with increasing v. In the case when $\theta^* = 0.8$, the value of the thermal saturation time is the highest; it decreases from a value of $\tau^* = 0.05$ s to $\tau^* = 0.003$ s at $v = 0.07$ m/s.

For saw teeth made of carbon steels with a thermal conductivity coefficient of $a = 8\cdot10^{-6}$ m^2/s, the thermal saturation time doubles at the corresponding values of θ^* (Figure 4b).

Figure 5 shows the change in thermal saturation time from the linear speed of the abrasive wheel during sharpening of the teeth at different values of the thermal conductivity coefficient a of the saw teeth material, and $\theta^* = 0.6$ in the feed speed from 0.015 m/s to 0.08 m/s.

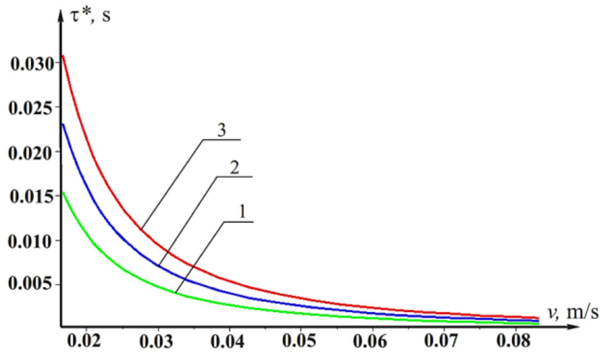

Figure 5 – Dependencies of the thermal saturation time on the linear speed of the abrasive wheel at $\theta^* = 0.6$: 1 – $a = 4\cdot10^{-6}$ m^2/s; 2 – $a = 6\cdot10^{-6}$ m^2/s; 3 – $a = 8\cdot10^{-6}$ m^2/s

Figure 5 shows the graphs of changes in τ^* in the range of abrasive wheel feed speeds from 0.067 m/s to 0.083 m/s. For saws made of steels with a thermal conductivity coefficient of $a = 4\cdot10^{-6}$ m^2/s (curve 1), the time of thermal saturation of the teeth during sharpening is almost half the time of thermal saturation of the saw teeth that are made of steel with a thermal conductivity coefficient of $a = 8\cdot10^{-6}$ m^2/s.

If it is necessary to reach $\theta^* = 0.6$, at a longitudinal feed rate of the abrasive wheel of $v = 0.083$ m/s, which corresponds to 5 m/min; thermal saturation of the cutting edge of the tooth occurs during a time interval equal to 1.2 ms for a saw made of carbon steel, and 0.6 ms for the saw made of alloy steel.

At a longitudinal feed rate of $v = 0.1$ m/s, which corresponds to 6 m/min, the thermal saturation of the cutting edge of the tooth occurs within an extremely short time period, which is 0.9 ms if the saw is made of carbon steel, and 0.4 ms for the saw made of alloy steel.

This two-fold discrepancy, depending on the material of the woodworking saw, should be taken into account when calculating the lengths of the cutting segments of abrasive wheels with gaps for sharpening woodworking saws made of steels with different thermal conductivity coefficients. Increasing the feed rate of the abrasive wheel from 5 m/min to 6 m/min causes a 1.4-fold increase in the thermal saturation time.

The results of calculating the lengths of the cutting segments using formula (28) and their number using formula (29) are summarized in Table 1.

Table 1 – Lengths of cutting segments and their number on the surface of an intermittent abrasive wheel with a diameter D = 300 mm and a rotation speed n = 3000 rpm

Longitudinal feed rate of the wheel v, m/s	The coefficient of thermal conductivity of the saw material a, m^2/s					
	$8 \cdot 10^{-6}$		$6 \cdot 10^{-6}$		$4 \cdot 10^{-6}$	
	Segment length l, mm	Number of segments N	Segment length l, mm	Number of segments N	Segment length l, mm	Number of segments N
Relative temperature θ^* = 0.2						
0.067	5.4	88	4.0	117	2.7	175
0.083	3.4	137	2.5	182	1.7	274
Relative temperature θ^* = 0.4						
0.067	28	17	21	22	14	33
0.083	18	26	14	35	9	52
Relative temperature θ^* = 0.6						
0.067	90	5	68	7	45	10
0.083	58	8	44	11	29	16

5 Discussion

As the analysis of modern works has shown, the quality of cutting tool sharpening, as well as the grinding of workpieces, is related to the thermal processes that occur during the interaction of the abrasive tool and the workpiece. Sharpening the cutting edges of the teeth of woodworking saws is performed at lower longitudinal feed rates of the abrasive wheel, compared to the process of grinding the teeth of wheels or bearing rings.

However, even at a low feed rate of the abrasive wheel (up to 5 m/min), burns, burrs, and bends of the tooth tips appear on the cutting edges of the saw teeth.

Therefore, it is advisable to use abrasive wheels with intermittent working surfaces for sharpening. The lengths of the cutting segments on the abrasive wheel and the number of segments depend on the blade material and the feed rate of the intermittent abrasive wheel.

By using such wheels, it is possible to achieve a predetermined temperature in the sharpening zone that does not exceed the thermal saturation temperature at which phase transformations occur in the saw material. The number of wheel cutting segments obtained in this paper at a relative temperature and a longitudinal feed rate of 5 m/min is in good agreement with the experimental data [1]. The authors investigated the patterns of influence of three factors: the number of gaps in the working surface of the wheel, the cutting depth, and the cutting speed on the temperature in the contact zone between the wheel and the surface of the saw tooth. A second-order regression equation was obtained.

Analysis of the regression equation shows that with a cutting depth of 0.3 mm and a cutting speed of 38 m/s during sharpening with a solid abrasive wheel with a diameter of 300 mm, the temperature in the contact zone reaches 728 °C, and during sharpening under the same conditions with wheels with 6–8 gaps, the temperature drops to 470 °C, which is 1.65 times lower than with a solid wheel.

Based on the experimental results, it was concluded that the rational number of gaps in a wheel is between 6 and 9, and the optimal number of gaps, considering design considerations, is eight, which aligns with the data presented in Table 1 above.

6 Conclusions

A solution to the three-dimensional Cauchy problem for thermal conductivity has been found, taking into account the process of sharpening sufficiently thin teeth of woodcutting saws and restrictions on the thermal conductivity coefficients of the saw material.

Based on the solution of the Cauchy problem for the differential thermal conductivity equation using the Fourier transform, a mathematical model of the thermal interaction between the abrasive wheel and the saw tooth during sharpening was built. The dependence for determining the time of thermal saturation of the saw tooth was obtained. It was found that the time of thermal saturation significantly depends on the longitudinal feed rate of the abrasive wheel and the thermal conductivity coefficient of the steel from which the wood machining saw is made.

Based on the developed mathematical model, the change in the time of thermal saturation of the saw tooth was investigated, depending on the feed rate of the abrasive wheel and the material of the woodcutting saw. The lengths of the cutting segments on the working surface of an intermittent abrasive wheel were determined, and the number of segments was calculated, taking into account the time required for thermal saturation of the saw tooth. The number of wheel cutting segments obtained at a relative temperature and a longitudinal feed rate of 5 m/min is in good agreement with the experimental data on temperature measurements at the cutting edge of a carbon steel frame saw tooth, confirming the reliability of the simulation results.

By using an intermittent abrasive wheel with the appropriate design parameters for specific sharpening conditions, a predetermined temperature in the sharpening zone is achieved that does not exceed the thermal saturation temperature, i.e., the temperature at which no phase transformations occur in the saw material.

As a result, the temperature on the surface of the saw tooth is reduced, which is the basis for increasing the productivity and quality of sharpening of woodcutting saws. These wheels are particularly effective when used for sharpening frame and circular steel saws, which are commonly used for cutting wood.

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