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WIMAN'S TYPE INEQUALITY IN THE UNIT BALL AND THE DIAGONAL MAXIMAL TERM

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Let $\mathbb{C}^p$ be the p-dimensional $(p \geq 1)$ a complex vector space, $\|n\| = n_1 + \cdots + n_p$, $|z| = \sqrt{|z_1|^2 + \ldots + |z_n|^2}$ for $n = (n_1, \ldots, n_p) \in \mathbb{Z}_+^p$ and $z = (z_1, \ldots, z_p) \in \mathbb{C}^p$, $\mathbb{R}_+ = [0, +\infty)$. In the paper we consider the class of analytic functions f, represented in the unit ball $\mathbb{B}_p = \{z \in \mathbb{C}^p \colon |z| < 1\}$ by power series of the form $f(z) = \sum\limits_{k=0}^{+\infty} P_k(z)$; here $P_0(z) \equiv a_0 \in \mathbb{C}$, $P_k(z)$ is a homogeneous polynomial of degree $k \in \mathbb{Z}_+$.
We denote $M_f(r) = \max\{|f(z)| \colon |z| = r\}$ and $m(r, f) = \max\{|P_k(z)| \colon k \geq 0\}$, the maximum modulus and maximal term of series, respectively; $r \in [0, 1)$. In particular, the following statements are proved:
1^0. *For every analytic function $f \in \mathscr{A}^p$, $p \geq 2$ and for any $\varepsilon > 0$ there exists a set $E = E(\varepsilon, f) \subset (0, 1)$ of finite logarithmic measure such that the*

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inequality $M(r,f) \le \frac{m(r,f)}{(1-r)^{1+\varepsilon}}\Big(\ln\Big(\frac{m(r,f)}{1-r}\Big)\Big)^{1/2+\varepsilon}$ *holds for all* $r \in (0,1)\setminus E$.
2^0. *Let* h *be a continuous positive increasing to* $+\infty$ *on* $[0;1)$ *functions such that* $\int_0^1 h(r)dr = +\infty$. *If the function* $f \in \mathscr{A}^p$ *is unbounded, then there exists a set* $E_1 := E(f,h) \subset (0,1)$ *such that* $\ln M(r,f) \le (1+o(1))\ln\big(h(r)m(r,f)\big)$ $(r \to 1-0,\ r \in (0,1)\setminus E_1)$, *and* h-meas $E_1 = \int_{E_1\cap(0,1)} h(r)dr < +\infty$.

Key words: *analytic function of several complex variables; diagonal maximal term; homogeneous polynomial.*

1. Introduction

We use the following standard notation. Let $\mathbb{C}^p$ be the p-dimensional $(p \ge 1)$ a complex vector space, $\mathbb{Z}^p_+ = (\mathbb{N}\cup\{0\})^p$, $z^n = z_1^{n_1}\cdots z_p^{n_p}$, $\|n\| = n_1+\cdots+n_p$, $|z| = \sqrt{|z_1|^2+\ldots+|z_n|^2}$ for $n = (n_1,\ldots,n_p) \in \mathbb{Z}^p_+$ and $z = (z_1,\ldots,z_p) \in \mathbb{C}^p$, $\mathbb{R}_+ = [0,+\infty)$. By $\mathscr{A}^p$ we denote of the class of analytic functions $f\colon \mathbb{B}^p \to \mathbb{C}$, (i.e., analytic functions of p complex variables in a ball $\mathbb{B}^p := \{z \in \mathbb{C}^p\colon |z| < 1\}$, represented by power series of the form

$$f(z) = \sum_{k=0}^{+\infty} P_k(z), \qquad z \in \mathbb{C}^p; \tag{1}$$

here $P_0(z) \equiv a_0 \in \mathbb{C}$, $P_k(z) = \sum_{\|n\|=k} a_n z^n$ is a homogeneous polynomial of degree $k \in \mathbb{Z}_+$.

From the one hand, it is well-known that every analytic function f in the complete Reinhardt domain $\mathbb{G}$ with center at $z = 0$ can be represented in $\mathbb{G}$ by the series of the form

$$f(z) = f(z_1,\ldots,z_p) = \sum_{\|n\|=0}^{+\infty} a_n z^n. \tag{2}$$

On the other hand, the domain of convergence of each series of form (2) is the logarithmically-convex complete Reinhardt domain with center $z = 0$.

A ball $\mathbb{B}_p(r) := \{z \in \mathbb{C}^p\colon |z| := \sqrt{|z_1|^2+\ldots+|z_p|^2} < r\}$, $r \in (0,+\infty)$, is the logarithmically-convex complete Reinhardt domain with center $z = 0$.

For $r > 0$, $k \ge 0$ and an entire function $f \in \mathscr{A}^p$ of the form (1) we denote

$$M(r,f) = \max\{|f(z)| : z \in \overline{\mathbb{B}}_p(r)\}, \quad m_k(r,f) = \max\{|P_k(z)| : z \in \overline{\mathbb{B}}_p(r)\}.$$

By the maximum modulus principle there exists a point $z^{(k)} \in \partial\mathbb{B}_p(r)$ such that $m_k(r,f) = |P_k(z^{(k)})|$. The definition of $\mathbb{B}_p(r)$ implies $z^{(k)}/r \in \partial\mathbb{B}_p$, hence, $P_k(z^{(k)}) = r^k P_k(s^{(k)})$. Thus $|P_k(s^{(k)})| = \max\{|P_k(z)|\colon z \in \overline{\mathbb{B}}_p\} = m_k(1, f)$ and therefore $s^{(k)}$ does not depend on r. So,

$$m_k(r, f) = r^k |P_k(s^{(k)})| \quad (r > 0,\ k \geq 0).$$

According to [1] define now the *diagonal maximal term of the series* (1)

$$m(r, f) \overset{def}{=} \max\{m_k(r, f) : k \geq 0\} = \max\{r^k m_k(1, f) : k \geq 0\}.$$

For entire functions of the form (1), analogues of the classical Wiman inequality were probably first established in the article [1]. In the article [2], analogues of the Bitlian-Goldberg inequality are established in the case where $\deg P_k = \lambda_k$ an increasing sequence of natural numbers is such that for the counting function $n(t) = \sum_{\lambda_k \leq t} 1$ the following condition holds

$$(\exists p_1 \in (0, +\infty))(\exists t_0 > 0)(\forall t \geq t_0)\colon \quad n(t + \sqrt{\psi(t)}) - n(t - \sqrt{\psi(t)}) \leq t^{p_1}$$

for some functions $\psi\colon \mathbb{R}_+ \to \mathbb{R}_+$ from the class $\mathscr{W}$ of positive continuous increasing on $[0, +\infty)$ functions such that

$$\int_{x_0}^{+\infty} \frac{dx}{\psi(x)} < +\infty$$

for some $x_0 \in (0, +\infty)$. In this article, we will prove for analytic functions $f \in \mathscr{A}^p$ of form (1) in the unit ball a number of analogues of the Wiman and Kövari inequalities, as well as the inequalities from paper [3].

Let $\mathscr{L}$ be a class of positive continuous on $\mathbb{R}_+ := [0, +\infty)$ the functions $l(t)$ such that $l(t) \to +\infty$ $(t \to +\infty)$. By $\mathscr{L}_+$ we denote the subclass of L such that $l(t) \uparrow +\infty$ as $x \to +\infty$.

2. Some estimates for integral of small parameter.

Let $\mathscr{I}_0(\nu)$ be the class of the functions $F\colon (-\infty, 0) \to \mathbb{R}_+$ defined by the Laplace-Stieltjes integral of the form

$$F(x) = \int_{\mathbb{R}_+} a(u) e^{xu} \nu(du), \tag{3}$$

where ν is a countable additive measure on the σ-algebra $\mathscr{B}(\mathbb{R}_+)$ of Borel sets on $\mathbb{R}_+$ (Borel measure) such that $\nu(\{x\colon 0 \leq x \leq b\}) < +\infty$ for any $b > 0$, and the function $a\colon \mathbb{R}_+ \to \mathbb{R}_+$ be a measurable function. Denote by supp ν the support of the measure ν, i.e. the closed set $E =\colon \operatorname{supp} \nu$ such that $\nu(\mathbb{R} \setminus E) = 0$ and $\nu(\{u \in \mathbb{R}\colon |u - u_0| < r\}) > 0$ for any $u_0 \in E$ and $r > 0$. For $x \in \mathbb{R}$ and $F \in \mathscr{I}(\nu)$ we set

$$\mu_*(x) = \sup\{a(u) e^{xu}\colon u \in \operatorname{supp} \nu\}, \quad \mu^*(x) = \sup\{a(u) e^{xu}\colon u \in \mathbb{R}\}.$$

Note that analogues of Wiman's inequality for Laplace-Stieltjes integrals were previously established, for example, in the articles [4–6].

Let $\mathscr{H}_0$ be the class of positive local integrable on $(-\infty,0)$ non-decreasing unbounded functions $h_0(x)$ such that $\int_{-1}^0 h_0(x)\,dx = +\infty$, $\mathscr{H}_1$ be the class of positive local integrable on $(0,1)$ non-decreasing unbounded functions $h_1(r)$ such that $\int_0^1 h_1(r)\,dr = +\infty$.

First, we formulate one statement from paper [3] containing an estimate of the exceptional set in the inequality for the derivative of functions from the class $\mathscr{I}_0(\nu)$.

We need the following auxiliary statements.

Lemma 1.1 ([3], Lemma 1). *Let $t_0 \in (-\infty,0)$ and $g_0(x)$ be positive differentiable non-decreasing on $(t_0,0)$ function, $\psi \in \mathscr{W}$, and $h_0(x)$ be positive local integrable on $[t_0;0)$ function such that $\int_{t_0}^0 h_0(x)\,dx = +\infty$. Then there exists a set $E_0 \subset [t_0;0)$ such that $\int_{E_0} h_0(x)\,dx < +\infty$ and*

$$\forall x \in [t_0;0)\setminus E_0 : \quad g_0'(x) \leqslant h_0(x)\psi(g_0(x)).$$

For a Lebesgue measurable set $E \subset (-\infty,0)$, its logarithmic measure is the quantity

$$m_{\ln}(E) := \int_{E\cap[-1,0)} \frac{dx}{|x|}.$$

Proposition 1.1 ([5]). *Let $F \in \mathscr{I}_0(\nu)$ and the following condition is satisfied*

$$(\exists c_1>0)(\exists c_2>0)(\forall a>0)(\forall b\in(0,a]) : \quad \nu(a-b,a+b] \le c_1 b + c_2. \tag{4}$$

Then for each $\varepsilon>0$ there exists a set $E\subset(-\infty;0)$ of finite logarithmic measure, that is, $m_{\ln}(E)<+\infty$, such that for every $x\in[-1,0)\setminus E$

$$F(x) \le \frac{\mu_*(x)}{|x|^{1+\varepsilon}}\Big(\ln\Big(\frac{\mu_*(x)}{|x|}\Big)\Big)^{1/2+\varepsilon}. \tag{5}$$

We prove now the following proposition.

Proposition 1.2. *Let $h\in\mathscr{H}_0$ be a function such that $h(x)\ge 1$ $(x\in[x_1,0))$ for some $x_1\in(-\infty,0)$. If $F\in\mathscr{I}_0(\nu)$, condition (4) is satisfied and $t_0\in(-\infty,0)$ be such that $\ln F(t_0)\ge u_0$, $u_0(\ln u_0)^2 = 2c_2$. Then, there exists a set $E_0 := E(F,h)\subset(-\infty,0)$ such that*

$$F(x) \le 2\mu_*(x)h_0(x)g(x)(\ln g(x))^2 \quad (\forall x\in[x_0,0)), \quad g(x)=\ln F(x), \tag{6}$$

for $x_0 = \max\{x_1,t_0\}$, and h_0-meas $E_0 := \int\limits_{E_0} h_0(x)dx < +\infty$.

$$(\forall\ x\in(t_0,0)\setminus E_0) : \ \ln F(x) \le 2\ln\big(h_0(x)\mu_*(x)\big)$$

From Proposition 1.2 we obtain the following corollary.

Corollary 1.1. *Let $h \in \mathscr{H}_0$, $F \in \mathscr{I}_0(\nu)$ and condition* (4) *is satisfied. If the function F is unbounded , then there exists a set $E_0 := E(F,h) \subset (-\infty,0)$ such that h_0-*meas $E_0 < +\infty$ *and*

$$\ln F(x) \le (1+o(1)) \ln \big(h_0(x)\mu_*(x)\big) \quad (x \to -0,\ x \in (-1,0) \setminus E_0), \tag{7}$$

in particular, $h_0(x)\mu_*(x) \to +\infty$ $(x \to -0)$.

Proof of Proposition 1.2. Denote $g(x) = \ln F(x)$. Then $g'(x) = F'(x)/F(x)$. Hence, for $a = 2g'(x)$ we obtain

$$F(x) - \int_{u \le a} a(u)e^{xu}\nu(du) = \int_{u>a} a(u)e^{xu}\nu(du) \le$$
$$\le \frac{1}{a}\int_{u>a} ua(u)e^{xu}\nu(du) \le \frac{F'(x)}{a} = \frac{1}{2}F(x).$$

Hence,

$$F(x) \le 2\int_{u \le a} a(u)e^{xu}\nu(du) \le 2\mu_*(x)\nu(2g'(x)).$$

But, by condition (4), $\nu(2g'(x)) \le c_1 g'(x) + c_2$, thus,

$$F(x) \le 2\mu_*(x)(c_1 g'(x) + c_2).$$

By Lemma 1.1 with $g_0(x) = g(x)$, $\psi(u) = u(\ln u)^2/c_1 - c_2/c_1$, there exist a set $E \subset (-\infty,0)$ such that $g'(x) \le h_0(x)\Big(g(x)(\ln g(x))^2/c_1 - c_2/c_1\Big)$ for all $x \in (t_0,0) \setminus E$ and $\int_E h_0(x)dx < +\infty$. Therefore,

$$F(x) \le 2\mu_*(x)\Big(h_0(x)g(x)(\ln g(x))^2 + c_2 - c_2 h_0(x)\Big).$$

Since, $h_0(x) \ge 1$ $(\forall x \in [x_1,0)$ for some $x_1 < 0$, thus $c_2 - c_2 h_0(x) \le 0$ $(\forall x \in [x_1,0))$ and

$$\forall x \in [x_0,0) \setminus E: \quad F(x) \le 2\mu_*(x)h_0(x)g(x)(\ln g(x))^2,$$

where $x_0 = \max\{x_1, t_0\}$. □

Proof of Corollary 1.1. From inequality (6) in Proposition 1.2, we obtain

$$\ln F(x) \le \ln 2 + \ln \Big(\mu_*(x)h_0(x)g(x)(\ln g(x))^2\Big) \quad (x \in [x_0,0) \setminus E). \tag{8}$$

But, the function F is unbounded. Therefore,

$$\ln 2 + \ln \Big(g(x)(\ln g(x))^2\Big) = o(\ln F(x)) \quad (x \to -0).$$

Applying this inequality in (8), we obtain asymptotic relation (13). □

3. Wiman's type inequality for analytic functions ion the unit ball and diagonal maximal term.

Based on Proposition 1.1, we will first prove the following statement.

Theorem 1.1. *For every analytic function $f \in \mathscr{A}^p$, $p \geq 2$ and for any $\varepsilon > 0$ there exists a set $E = E(\varepsilon, f) \subset (0,1)$ of finite logarithmic measure, i.e.,* $\ln-\text{meas}\, E = \int_{E\cap(0,1)} \frac{dr}{1-r} < +\infty$, *such that the inequality*

$$M(r,f) \leq \frac{m(r,f)}{(1-r)^{1+\varepsilon}} \Big(\ln\Big(\frac{m(r,f)}{1-r}\Big)\Big)^{1/2+\varepsilon} \tag{9}$$

holds for all $r \in (0,1)\setminus E$.

Proof of Theorem 1.1. Let (P_k) be a homogeneous polynomial of $\deg P_k = k$, $z = rs = (rs_1, \dots, rs_p) \in \mathbb{B}_p(r)$, $0 < r < 1$. Hence, $s \in \mathbb{B}_p$. So, we have $f(z) = \sum_{k=0}^{+\infty} r^k P_k(s)$. Since, $m_k(1,f) = |P_k(s^{(k)})| = \max\{|P_k(z)|\colon z \in \overline{\mathbb{B}}_p\}$, we obtain

$$\begin{aligned} M(r,f) &= \max\{|f(z)|\colon z \in \overline{\mathbb{B}}_p(r)\} = \max\{|f(rs)|\colon s \in \overline{\mathbb{B}}_p\} \leq \\ &\leq \sum_{k=0}^{+\infty} r^k \max\{|P_k(s)| : s \in \overline{\mathbb{B}}_p\} = \sum_{k=0}^{+\infty} r^k m_k(1,f) \overset{def}{=} H(r). \end{aligned} \tag{10}$$

Let $\nu(E) = \sum\limits_{k=0}^{+\infty} \delta_k(E)$, where $\delta_\lambda(E) = 1$ for $\lambda \in E$, $\delta_\lambda(E) = 0$ for $\lambda \notin E$ for every bounded set $E \subset \mathbb{R}_+$. We put $a(u) = m_k(1,f)$ for $u = k$, $a(u) = 0$ for $u \in \mathbb{R}_+ \setminus \mathbb{Z}_+$.

Then

$$F(x) := H(e^x) = \int_{\mathbb{R}_+} a(u) e^{ux} \nu(du), \quad \mu_H(e^x) = \mu_*(x) \quad (x < 0). \tag{11}$$

Here F is a function, for which condition (4) of Proposition 1.1 holds. We put $r = e^x$ and denote $E = e^{E_1}$. Now from Proposition 1.1 it follows

$$\begin{aligned} M(r,f) = M(e^x,f) = F(x) &\leq \frac{\mu_*(x)}{|x|^{1+\varepsilon}} \Big(\ln\Big(\frac{\mu_*(x)}{|x|}\Big)\Big)^{1/2+\varepsilon} = \\ = \frac{\mu_H(e^x)}{|x|^{1+\varepsilon}} \Big(\ln\Big(\frac{\mu_H(e^x)}{|x|}\Big)\Big)^{1/2+\varepsilon} &= \frac{m(r,f)}{|x|^{1+\varepsilon}} \Big(\ln\Big(\frac{m(r,f)}{|x|}\Big)\Big)^{1/2+\varepsilon}. \end{aligned} \tag{12}$$

for all $x \in (-\infty, 0)\setminus E_1$, $m_{\ln}(E_1) := \int_{E_1\cap[-1,0)} \frac{dx}{|x|} < +\infty$.

It is easy to see that $1 - r < |x|$. So, from inequality (12) we obtain that inequality (9) holds for all $r \in (0,1)\setminus E$. Since, $|\ln r| \sim 1-r \sim |x|$ $(r \to 1-0)$ and

$$\int_{E\cap(0,1)} \frac{dr}{1-r} < +\infty \Longleftrightarrow \int_{E\cap(1/e,1)} \frac{dr}{r(1-r)} = \int_{E\cap(1/e,1)} \frac{d\ln r}{1-r} < +\infty \Longleftrightarrow$$
$$m_{\ln}(E_1) = \int_{E_1\cap(-1,0)} \frac{dx}{|x|} = \int_{E\cap(1/e,1)} \frac{d\ln r}{|\ln r|} < +\infty.$$

So, $\ln -\text{meas } E = \int_{E\cap(0,1)} \frac{dr}{1-r} < +\infty$. □

In exactly the same way as we obtained Theorem 1.1, from Corollary 1.1 we obtain the following statement.

Proposition 1.3. *Let $h \in \mathscr{H}_1$, $f \in \mathscr{A}^p$. If the function f is unbounded, then there exists a set $E_1 := E(f,h) \subset (0,1)$ such that*

$$\ln M(r,f) \le (1+o(1))\ln\big(h(r)m(r,f)\big) \quad (r \to 1-0,\ r \in (0,1)\setminus E_1), \tag{13}$$

and h-meas $E_1 = \int_{E_1\cap(0,1)} h(r)dr < +\infty$. In particular, $h(r)m(r,f) \to +\infty$ $(r \to 1-0,\ r \notin E_1)$.

Proof of Proposition 1.3. From asymptotic relation (13) in Corollary 1.1 with $h_0(x) = h(e^x)$, using (10) and (11), at $r = e^x$ we obtain

$$\begin{aligned} \ln M(r,f) &= \ln M(e^x,f) = \ln F(x) \le (1+o(1))\ln\big(h_0(x)\mu_*(x)\big) = \\ &= (1+o(1))\ln\big(h_0(x)\mu_H(e^x)\big) \quad (x \to -0,\ x \in (-1,0)\setminus E_0). \end{aligned} \tag{14}$$

Denote again $E_1 = \exp\{E_0\}$. Then from asymptotic relation (14) we obtain

$$\ln M(r,f) \le (1+o(1))\ln\big(h(r)\mu(r,f)\big) \quad (r \to 1-0,\ r \in (0,1)\setminus E_1),$$

and also

$$\begin{aligned} h-\text{meas}(E_1) &= \int_{E_1} h(r)dr \le \int_{E_1} h(r)d\ln r = \int_{E_0} h(e^x)dx = \\ &= \int_{E_0} h_0(x)dx = h_0 - \text{meas}(E_0) < +\infty. \end{aligned}$$

□

The following statement indicates that ε in the exponents of the power in the inequality (5) cannot, generally speaking, be replaced even by 0.

Proposition 1.4 ([5]). *For each measure ν such that*

$$(\forall x \in (-\infty, 0)): \qquad \int_{\mathbb{R}_+} \exp\{ux\}\nu(du) < +\infty$$

and

$$(\exists c_1 > 0)(\exists c_2 > 0)(\forall a > 0)(\forall b \in (0, a)): \quad \nu(a-b, a+b] \geq c_1 b + c_2, \tag{15}$$

there exists a function $F \in \mathscr{I}_0(\nu)$ such that

$$\varliminf_{x \to -0} F(x) \left(\frac{\mu_*(x)}{|x|} \left(\ln \left(\frac{\mu_*(x)}{|x|} \right) \right)^{1/2} \right)^{-1} > 0. \tag{16}$$

Remark 1.1. It seems plausible that from this statement we can obtain a similar statement for analytic functions in the unit ball. We add that we managed to implement a similar approach in paper [2] for entire functions of form (1).

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НЕРІВНІСТЬ ТИПУ ВІМАНА І ДІАГОНАЛЬНИЙ МАКСИМАЛЬНИЙ ЧЛЕН

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Нехай $\mathbb{C}^p$ *–* p*-вимірний* ($p \geq 1$) *комплексний векторний простір,* $\|n\| = n_1 + \cdots + n_p$, $|z| = \sqrt{|z_1|^2 + \ldots + |z_n|^2}$ *для* $n = (n_1, \ldots, n_p) \in \mathbb{Z}_+^p$ *та* $z = (z_1, \ldots, z_p) \in \mathbb{C}^p$, $\mathbb{R}_+ = [0, +\infty)$. *В статті розглядається кла аналітичних функцій* f, *представлених в одиничній кулі* $\mathbb{B}_p = \{z \in \mathbb{C}^p \colon |z| < 1\}$ *степеневим рядом вигляду* $f(z) = \sum\limits_{k=0}^{+\infty} P_k(z)$; *тут* $P_0(z) \equiv a_0 \in \mathbb{C}$, $P_k(z)$ *– однорідний поліном степеня* $k \in \mathbb{Z}_+$. *Позначимо* $M_f(r) = \max\{|f(z)| \colon |z| = r\}$ *і* $m(r, f) = \max\{|P_k(z)| \colon k \geq 0\}$ *– максимуум модуля і максимальний член ряду, відповідно. В статті, зокрема, доведено наступні твердження:*
1^0. *Для кожної аналітичної функції* $f \in \mathscr{A}^p$, $p \geq 2$ *і для довільного* $\varepsilon > 0$ *існує множина* $E = E(\varepsilon, f) \subset (0,1)$ *скінченної логарифмічної міри і така, що нерівність* $M(r, f) \leq \frac{m(r,f)}{(1-r)^{1+\varepsilon}} \left(\ln\left(\frac{m(r,f)}{1-r}\right)\right)^{1/2+\varepsilon}$ *виконується для всіх* $r \in (0,1) \setminus E$.
2^0. *Нехай* h *– неперервна додатна зростаюча до* $+\infty$ *на* $[0;1)$ *функція така, що* $\int_0^1 h(r)dr = +\infty$. *Якщо функція* $f \in \mathscr{A}^p$ *– необмежена, то існує множина* $E_1 := E(f, h) \subset (0,1)$ *така, що* h-meas $E_1 = \int_{E_1 \cap (0,1)} h(r)dr < +\infty$ *та* $\ln M(r, f) \leq (1 + o(1)) \ln\big(h(r) m(r, f)\big)$ $(r \to 1-0,\ r \in (0,1) \setminus E_1)$.

Ключові слова: *аналітична функція від багатьох комплексних змінних, діагональний максимальний член, однорідний поліном.*