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Modeling of Boundary-Value Problems of Heat Conduction for Multilayered Hollow Cylinder

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Abstract

The paper proposes the solution of the boundary value problem for the distribution of a non-stationary temperature field along the thickness of a multilayer hollow cylinder. The basis of the solution is the reduction method, the quasi-derivative concept, the modified Fourier method, and the problem of eigenvalues. The obtained analytical solution is modeled as a pseudocode and implemented on a specific numerical example.

REQUEST OF THE PROBLEM AND ITS MATHEMATICAL MODEL

The problem of the distribution of a non-stationary temperature field in the thickness of a hollow cylinder is reduced to a solution on the interval $[r_0 \neq 0, r_n]$ of the differential equation:

$$c\rho\frac{\partial t(r,\tau)}{\partial\tau}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial t(r,\tau)}{\partial r}\right), \qquad (1)$$

with the Newton's boundary condition

$$\begin{cases}\alpha_0 r_0 t(r_0,\tau)-t^{[1]}(r_0,\tau)=\alpha_0 r_0\psi_0(\tau),\\ \alpha_n r_n t(r_n,\tau)+t^{[1]}(r_n,\tau)=\alpha_n r_n\psi_n(\tau),\end{cases} \qquad (2)$$

and the initial condition

$$t(r,0)=\varphi(r) \qquad (3)$$

where $t(r,\tau)$ – is a temperature, ^{0}C; $t^{[1]}(r,\tau)=r\lambda t'_r$ – denotes a quasi-derivative; α_0 and α_n – coefficients of heat exchange between the fire medium and the cylinder surfaces, W/m$^2\cdot{}^0$C; $\psi_0(\tau)=345\lg((8\tau/60)+1)+20$ – the law of changing the temperature of the fire medium (standard temperature mode); $\psi_0(\tau)=20$ – temperature of the medium from the inside of the cylinder; τ – time, s.

COMPUTER SIMULATION

To solve the problem, we will conduct a computer simulation of a pseudocode solution for the computer algebra system

1) assign the values of the corresponding coefficients and notations of the problem: r_0:=0.2: r_n:=0.5: c:=840: ρ:=2400: λ:=2.5: α_0:=20: α_n:=4: ψ_0:=τ→(345·log10(8·τ+1)+20): ψ_n:=τ→20: φ:=x→20:

$\alpha(\omega):=\sqrt{\frac{c\cdot\rho\cdot\omega}{\lambda}}$: $\alpha(\omega_i):=\sqrt{\frac{c\cdot\rho\cdot\omega_i}{\lambda}}$:

2) assign the values of the corresponding matrices and vector: Br:=Matrix([[1, (lnr-lnr)/λ],[0,1]]):

B:=Matrix([[1,($\ln r_n-\ln r_0$)/λ],[0,1]]):

P:=Matrix([[$\alpha_0 r_0$, -1],[0,0]]):

Q:=Matrix([[0, 0],[$\alpha_n r_n$,1]]):

$$\beta_{11}=\frac{\pi\lambda\alpha r_0\left(J_1(\alpha r_0)\cdot N_0(\alpha r_n)-J_0(\alpha r_n)\cdot N_1(\alpha r_0)\right)}{2\lambda}:\quad \beta_{12}=\frac{\pi\left(J_0(\alpha r_0)\cdot N_0(\alpha r_n)-J_0(\alpha r_n)\cdot N_0(\alpha r_0)\right)}{2\lambda}:$$

$$\beta_{21}=\frac{\pi\lambda^2\alpha^2 r_n r_0\left(J_1(\alpha r_n)\cdot N_1(\alpha r_0)-J_1(\alpha r_0)\cdot N_1(\alpha r_n)\right)}{2\lambda}:\quad \beta_{22}=\frac{\pi\lambda\alpha r_0\left(J_1(\alpha r_n)\cdot N_0(\alpha r_0)-J_0(\alpha r_0)\cdot N_1(\alpha r_n)\right)}{2\lambda}:$$

Bω:=Matrix([[β_{11}, β_{12}],[β_{21},β_{22}]]):

Γ:=Vector([$\alpha_0 r_0\psi_0$(τ), $\alpha_n r_n\psi_n$(τ)]):

3) find the initial vector: P_0:=P+Q·B:

4) build the function u(r,τ): u:=(r,τ)→(1,0)·B·P_0:

5) find the characteristic equation of the problem for eigenvalues

Φ:=(ω)→(det(P+Q·Bω)):

6) find the eigenvalues ω_k: ω0:=0: **for** i **from** 1 **to** k **do** ω_i:=NextZero(ω→Φ(ω), ω_{i-1}) **end do:**

7) assign the values of corresponding matrices $B\omega_i$: **for** i **from** 1 **to** k **do** $B\omega_i$:=Matrix([[β_{11}, β_{12}],[β_{21},β_{22}]]) **end do:**

8) assign the values of corresponding eigenvalues: for i **from** 1 **to** k **do**

$$R_i:=c\cdot\rho\int_{r_0}^{r_n}(1\ \ 0)\cdot B\omega_i\cdot\begin{pmatrix}1\\ \alpha_0 r_0\\ 1\end{pmatrix}dr$$ **end do:**

9) the norm square of the eigenvalues: for i **from** 1 **to** k **do** $||R_i||:=c\cdot\rho\int_{r_0}^{r_n}(R_i)^2\,r\,dr$ **end do:**

10) the Fourier coefficients for development of the initial condition: for i **from** 1 **to** k **do**

$$f_i:=\frac{c\cdot\rho}{||R_i||}\int_{r_0}^{r_n}(\varphi(r)-u(r,0))\cdot R_i\,r\,dr$$ **end do:**

11) the Fourier coefficients for expansion of the function u(r,τ): for i **from** 1 **to** k **do**

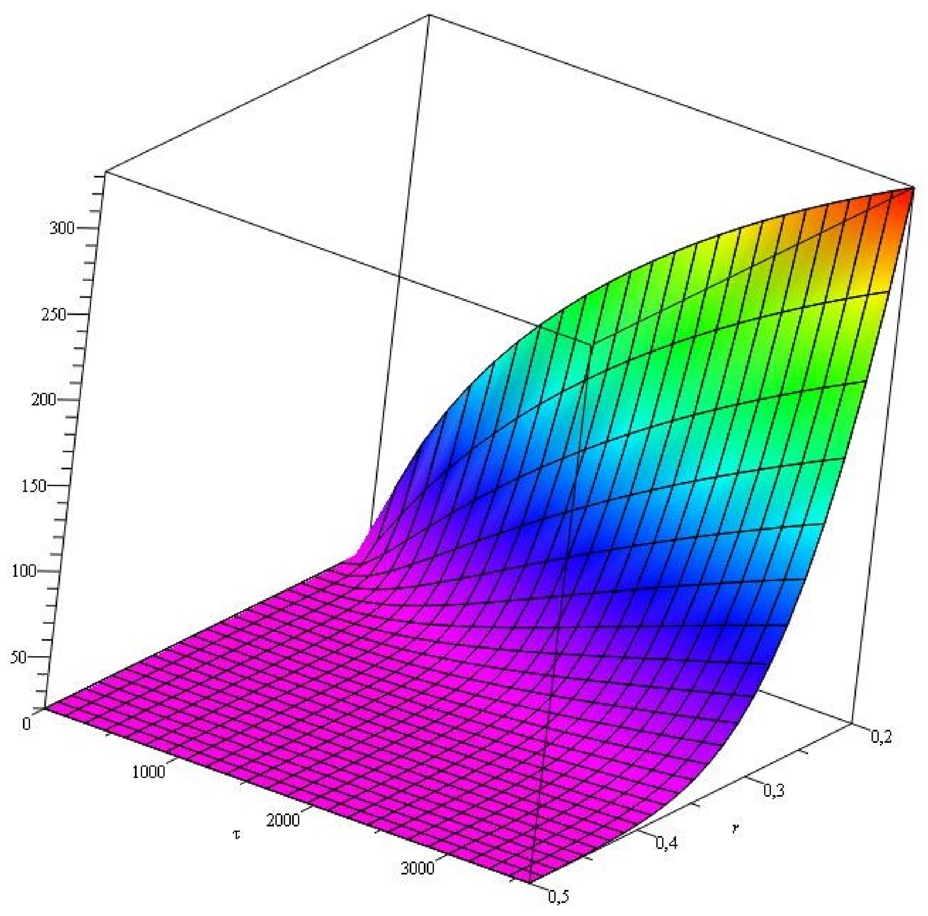
$$u_i:=\tau\to\frac{c\cdot\rho}{||R_i||}\int_{r_0}^{r_n}\left(\frac{\partial}{\partial\tau}(u(r,\tau))\right)\cdot R_i\,r\,dr$$ **end do:**

12) construction of the function v(r,τ): for i **from** 1 **to** k **do** $v_i:=\left(f_i\cdot e^{-\omega_i\tau}-\left(\int_0^\tau e^{-\omega_i(\tau-s)}u_i(s)\,ds\right)R_i\right)$ **end do:**

13) construction of the solution t(r,τ):

$$t:=(r,\tau)\to u(r,\tau)+\sum_{i=1}^{k}v_i:$$

14) result output in the form of the 3D-graph: plot3d(t(r, τ), r=r_0...r_n, τ=0..3600)



The results of calculations can also be displayed in the form of explicit formulas, graphs, animations, tables, etc.

Conclusion

The use of information technologies and methods of mathematical and computer modeling enables to quickly and clearly model certain phenomena or processes that arise during certain scientific researches. Using this approach significantly accelerates the time of research (compared with experimentally) and reduces the cost of scientific research.

The proposed direct method for calculating the temperature field in the plane structure can be used to study non-stationary thermal processes without the use of approximate and operational methods. Implementation of the results of such research has significantly reduced the time and amount of computing in determining the temperature distribution in plane structures, as well as improve the accuracy of the calculation in comparison with the approximate methods.