Application of Fractional Order Transfer Functions in Approximation of Voltage Divider With Supercapacitor Model

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***Abstract*— A complex fractional order transfer function of the voltage divider on the basis of the supercapacitor has been approximated by a particle swarm optimization method using a number of simple fractional order transfer functions proposed, and such approximation accuracy has been estimated through comparison of transition and frequency characteristics. The advantages of approximating a complex transfer function of the voltage divider based on the supercapacitor using the fractional order transfer functions have been shown. A comparative analysis of the results obtained for the different transfer functions has enabled drawing the conclusion about high degree of their coincidence and the possibility of their application for further studies.**

Keywords—supercapacitor, approximation, fractional order transfer function, particle swarm optimization method, transition function, voltage divider

I. INTRODUCTION

Supercapacitors, also referred to as ultracapacitors or ionistors, are the special two-layer electrochemical capacitors on the basis of organic or inorganic electrolyte, which, compared to conventional "dry" or electrolytic capacitors, have a double electrode layer at the border of the electrolyte. By its characteristics, a supercapacitor is located between a conventional capacitor and a chemical current source, such as a battery.

The thickness of the double electric layer in the supercapacitor is very small, which means that the energy stored in it is much higher in comparison with conventional capacitors. An important feature of the supercapacitor is the fact that the use of a double electric layer can significantly increase the electrode surface area, compared to the conventional capacitor dielectric. The specific capacity of the supercapacitor reaches tens of F/cm^3, at a nominal voltage of just a few volts. That is, supercapacitors are characterized by the high value of power density, low losses, durability, the ability to withstand a large number of charge / discharge cycles, relatively small dimensions, simplicity of the charging process.

The development of supercapacitors is now taking place in two main areas: high-capacity supercapacitors for transport and industry, in particular for the rapid energy accumulation while braking an electric car or any other electric vehicle and its delivery during acceleration, and supercapacitors of relatively small capacity for a variety of computer hardware technology, robots, mobile phones, camcorders, toys, etc. In the field of microelectromechanical systems (MEMS), there is a great need for the creation and application of compact high-capacity supercapacitors as micro-energy sources for low-power drives of various devices and sensors [1].

The increasing interest in the use of supercapacitors has necessitated the design and modelling of such systems, from which followed the need to create their models. Initially, a simple integer model (Fig. 1a) and the equivalent supercapacitor model, based on the combination of RC-elements (Fig. 1b) were proposed [2,3]. Such models were investigated and their advantages and disadvantages were identified. Further, due to the development of fractional calculus and the need to improve the models accuracy, this calculus was increasingly used to create new models of supercapacitors (Fig. 1c) and to improve the old ones (Fig. 1b).

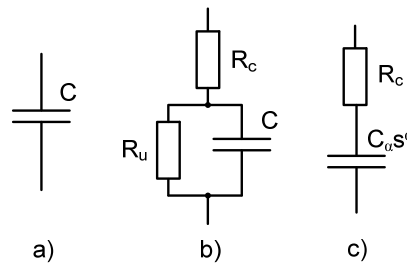

Fig. 1. Models of supercapacitors: a) ideal supercapacitor; b) equivalent supercapacitor model based on series and parallel resistances; c) equivalent supercapacitor model using fractional calculus

The analysis of the physical processes of the capacitors operation revealed that typical equivalent models of supercapacitors, containing one or two combined RC-elements parameters and constructed on the basis of integer-order equations, are not sufficient for accurate modelling of supercapacitors in dynamic working modes [4,5]. Therefore, complex equivalent supercapacitor models with many connected RC-elements or simpler models (Fig. 1b) described by fractional differential equations are used to solve this problem [4].

Fractional order calculus and dielectric relaxation models are often used to describe the impedance of supercapacitors [4,5]. The famous Debye model of ideal dielectric relaxation is practically replaced by its empirical modification, i.e. Havriliak-Negami model of complex dielectric constant

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{(1 + (j\omega T)^\delta)^\gamma}, \qquad (1)$$

where: ε_∞ – "infinite frequency" dielectric constant,
ε_s – "static frequency" dielectric constant,
ω – angular frequency,
T – time dielectric constant.

For $\gamma = 1$ equation (1) is converted into Cole-Cole equation

$$\varepsilon_{CC}(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + (j\omega T)^\delta}, \quad (2)$$

where $0 < \delta \le 1$.

For $\delta = 1$ equation (1) is converted into Cole-Davison equation

$$\varepsilon_{CD}(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{(1 + j\omega T)^\gamma}, \quad (3)$$

where $0 < \gamma \le 1$.

Fractional order calculus and a dielectric relaxation model which can be described by several of the above-mentioned models are used to describe supercapacitor impedance [4]. However, Cole-Cole model is proposed for modelling in the field of automation, although its transfer function (TF) is still rather difficult to investigate.

Another research highlights some practical applications of fractional order calculus in the field of supercapacitor modelling using fractional-order transfer functions based on Cole-Davidson model [5], which proves that they are more accurate than the previously applied integer RC-circuits, but are very complex for further research.

Thus, it can be concluded that there is a need to simplify the results obtained to facilitate further application and research.

When optimizing automatic control systems (ACS) for electromechanical systems (EMS) or MEMS with control objects described by high-order TFs, the researchers tend to often simplify such TFs and reduce the order of their numerator and denominator [6]. It was proven that representation of control objects using a simpler fractional order TF provides a reduction in the order of the output TF and can be effectively applied to approximate EMS and MEMS objects [7]. Thus, according to the developed approach, a new strategy for the approximation of high-order EMS and MEMS transfer functions with the use of simple fractional-order TF models without zeros was proposed [7]:

$$W(s) = \frac{k}{a_1 s^{\alpha_1} + 1}, \quad (4)$$

$$W(s) = \frac{k}{a_2 s^{\alpha_2} + a_1 s^{\alpha_1} + 1}. \quad (5)$$

This approach was further extended and developed in [8], where the approximation of high-order TFs was carried out by using simple fractional order TF models with zero and pole

$$W(s) = k\frac{b_1 s^{\beta_1} + 1}{a_1 s^{\alpha_1} + 1}. \quad (6)$$

A high-order TF approximation with the use of sufficiently simple and compact fractional order TFs in a given frequency range provides better results than traditional approaches to lower the order through integer order TF.

The purpose of this paper is to conduct further research for the development of a more comprehensive theory and to obtain new results in describing complex fractional order TFs of supercapacitor impedance or the voltage divider on its basis by applying simpler fractional order models, including the ones with zero and pole. In addition, these studies are relevant for the approximation of complex fractional order TFs obtained from the experimental identification of real supercapacitors by means of simpler fractional order TFs, as well as for the comparative analysis of such approximation accuracy in time and frequency domains. The results of the analysis offer an insight and some recommendations for further application of fractional order TFs under investigation to approximate complex fractional order TFs of the supercapacitor impedance or a divider on its basis by investigating their frequency and transition characteristics.

The main tasks of this paper are as follows:

- approximation of complex fractional order TF, which is the TF of a voltage divider created on the basis of a supercapacitor by using simpler fractional order models, and estimation of such approximation accuracy;

- comparison of approximation accuracy estimation of all fractional order TFs, both in time and frequency domains, for final confirmation of accuracy results.

II. SUPERCAPACITOR DESCRIPTION USING FRACTIONAL ORDER TF

The formula for determining the real impedance of a supercapacitor can be written on the basis of one of the above equations (1) - (3) [4], as well as by means of using the parameters of the equivalent supercapacitor model, shown in Fig. 1b, where R_u is parallel leakage resistance, and R_c is series equivalent resistance. Application of Cole-Cole equation (2) to obtain the capacity $C(j\omega) = C_0 \varepsilon_{CC}(\omega)$ has enabled to write the formula for determining the real impedance of the supercapacitor

$$Z_s(s) = R_c + \frac{R_u \dfrac{1}{j\omega C(j\omega)}}{R_u + \dfrac{1}{j\omega C(j\omega)}}.$$

The scheme consists of an operational amplifier, a R resistor and a supercapacitor of different capacities under investigation. In this scheme, a voltage divider is formed by a supercapacitor and active R resistor.

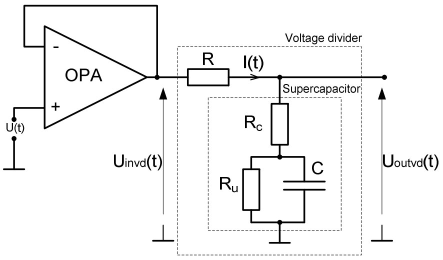

Fig. 2. The scheme used in experiments with supercapacitors

The TF of such voltage divider is generally described by the equation

$$W_d(s) = \frac{W_{zcc}(s)}{R_u + W_{zcc}(s)}. \quad (10)$$

Thus, there are two ways of obtaining simpler TFs of a voltage divider with a supercapacitor.

Variant 1. It is possible to approximate TF of the supercapacitor (9) using a simpler fractional order TF (4), whose the parameters can be determined, for example, by the PSO method.

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Variant 2. Substitution of the obtained expression (9) for (10) resulting in the form (8) enables to obtain the final expression of the TF of the voltage divider based on the supercapacitor

$$W_{vd2}(s) = \frac{b_5 s^2 + b_4 s^{1+\delta} + b_3 s^{2\delta} + b_2 s + b_1 s^{\delta} + b_0}{a_5 s^2 + a_4 s^{1+\delta} + a_3 s^{2\delta} + a_2 s + a_1 s^{\delta} + a_0}, \quad (12)$$

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III. Particle Swarm Optimization Method

The peculiarity of the PSO method is N of particles moving in the D-dimensional search space, where each particle is characterized by random position and velocity parameters. Each particle changes its trajectory in space, depending on its own and group experience gained at each iteration.

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IV. Approximation of Complex TF of Supercapacitor Using Simple Fractional Order TFs

Approximation of high order part of voltage divider with supercapacitor with TF (13) has been carried out by means of using fractional order TFs (4) - (6), whose parameters have been found through PSO method.

By applying MATLAB software environment, a necessary computer program has been developed to approximate high-order TFs using quite simple fractional order TFs (4) - (6), including the ones with zero and pole, on the basis of particle swarm optimization method with a minimal set error. This approach allows to look for the best options of parts approaching, particularly in EMS or MEMS in accordance with one-to- five unknown parameters [7,8].

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V. Conclusions

1. The expression of the fractional order transfer function of the voltage divider built on the basis of a supercapacitor for its equivalent model which is used for experimental studies has been analytically obtained. Two ways of obtaining simpler transfer functions of a voltage divider with a supercapacitor have been considered.

2. Complex fractional order transfer function of the voltage divider with a supercapacitor has been approximated by means of applying the proposed relatively simple fractional order transfer functions and particle swarm optimization method.

3. It has been verified that this approach to approximating a complex fractional order transfer function of a voltage divider with a supercapacitor is much more advantageous from the point of view of describing the control objects that were obtained as a complex fractional order transfer function after object identification.

4. According to the results of the comparative analysis, it has been proven that the application of all three proposed simple fractional order transfer functions resulted in a very high degree of coincidence of transition characteristics with an error of no more than 0.35%, and a high degree of coincidence in the low frequency range in the Bode diagrams. The best degree of coincidence in the Bode diagrams was demonstrated by the fractional order transfer function with zero and pole (6).

5. The use of zero and pole fractional order transfer function (6) is recommended for the approximation of voltage divider with supercapacitor, since it provides higher frequency approximation accuracy than the other proposed fractional order transfer functions.

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