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DIRECT (CLASSICAL) METHOD OF CALCULATION OF THE TEMPERATURE FIELD IN A HOLLOW MULTILAYER CYLINDER

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A structural scheme of construction of a solution to a mixed problem for the heat-conduction equation in the case of a hollow multilayer cylinder under the conditions of ideal thermal contact between the layers is proposed and substantiated. The presence of connective heat exchange with the environment is assumed, i.e., boundary conditions of the third kind are observed. Coefficients of the heat-conduction equation are considered to be piecewise-continuous as far as the space coordinate is concerned. The scheme is based on the: reduction method, concept of quasi derivatives, modcern theory of systems of linear differential equations, Fourier method, and modified method of eigenfunctions. A numerical example of calculation of the temperature field in an actual hollow four-layer pipeline under external-fire conditions is given.

Keywords: *reduction, quasi derivative, Bessel function, Cauchy matrix, method of eigenfunctions.*

Introduction. Analytical methods of calculation of temperature fields in inhomogeneous laminar structures are the focus of voluminous literature. In particular, in the case of a hollow multilayer cylinder, until the present time, use has been made of Laplace and Fourier transformation methods and of Green′s functions (e.g., [1–3]).

In [4], we have proposed and substantiated a structural scheme to construct a solution to a mixed problem for the heat-conduction equation with piecewise-continuous coefficients dependent on the space coordinate on a finite interval. The scheme was based on the: reduction method, concept of quasi derivatives, modern theory of systems of linear differential equations, Fourier method, and modified method of eigenfunctions. It is our opinion that the advantage of this approach enables one to, at least, avoid the procedure of finding inverse transforms which presents the greatest difficulty when integral-transformation methods are employed [5].

The obtained results can be used, e.g., in investigating the process of heat transfer in a multilayer slab under the conditions of ideal thermal contact between the layers. This idea has already been implemented in [6–8]. Clearly, the question arose as to the extension of this scheme to the case of multilayer structures of other canonical forms.

In the present paper, we propose a scheme of solution of a mixed problem for the heat-conduction equation with piecewise-linear coefficients that arises when investigating the temperature field in a hollow multilayer cylinder under the conditions of ideal thermal contact with the layers. To simplify calculations, we consider boundary conditions of the third kind on interior and exterior surfaces. We consider, as an example, the problem of propagation of the temperature field inside a four-layer pipeline under external-fire conditions.

Formulation of the Problem. An unbounded hollow n layer cylinder with radii $r_0, r_1, r_2, \ldots, r_{n-1}, r_n$ is given, with $0 < r_0 < r_1 < r_2 < \ldots < r_{n-1} < r_n$ (Fig. 1), and an initial temperature distribution $\varphi(r)$ is assigned. The cylinder temperature depends on the radius r and the time τ. An isotropic material of thermal-conductivity coefficient λ_i, specific heat c_i, and density ρ_i is taken to fabricate the ith layer ($i = 0, 1, 2, \ldots, n - 1$). We assume that on the interior ($r = r_0$) and exterior ($r = r_n$) surfaces, there is convective heat exchange with the environment, i.e., boundary conditions of the third kind are observed. The coefficients of heat transfer of the interior and exterior surfaces are dissimilar ($\alpha_0 \neq \alpha_n$) [5, 9].

Let θ_i be the characteristic function [10, 11] of the interval $[r_i, r_{i+1})$, i.e.,

$$\theta_i(r) = \begin{cases} 1\,, & r \in [r_i, r_{i+1})\,, \\ 0\,, & r \notin [r_i, r_{i+1})\,, \end{cases} \quad i = \overline{0, n-1}\,.$$

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