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CONTENTS

Approximation of PM DC Micromotor Transfer Function By Fractional Order Transfer Function

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***Abstract*— Transfer function of permanent magnets (PM) DC micromotor speed is quite complex, which in turn complicates the synthesis of automatic control systems. It is established that for this class of micromotors the electromechanical time constant becomes commensurate with the electric time constant of the armature, which leads to overshooting of the output coordinate, so simplification of transfer function of speed by discarding the s^2 component will result in aperiodic process and considerable accuracy loss. A series of experimental PM DC micromotor studies using a high-precision laser tachometer to obtain the necessary parameters for the high-accuracy micromotor model developement have been carried out. The developed high- accuracy model made it possible to obtain the appropriate transition process of speed with overshooting. Approximation of complex transfer function of PM DC micromotor speed by a more compact fractional order transfer function using the intelligent particle swarm optimization method has been performed, and the approximation accuracy has been estimated on the basis of transition and frequency characteristics analysis.**

Keywords*— *fractional order, transfer function, permanent magnets DC micromotor, approximation, identification

I. INTRODUCTION

Recent developments in permanent magnets (PM) DC micromotors research [1,2,3] have shown a growing interest in this class of motors, in particular due to the widespread application of this motor type in various miniature physical models, hand tools, homemade devices, micro machining stations, micro actuator drives using various gearboxes and quadcopters.

PM DC micromotors have been popular for a long time. This popularity is based on their characteristic strengths - they are simple, compact, light, high-speed and provide good starting and operating torque. For their operation it is enough to use the battery or the accumulator of the appropriate supply voltage, but it results in unregulated speed of rotation. In addition, there is a large number of ready-made technical means for smooth speed control in a wide range from 0 to maximum, such as PWM controllers. If necessary, these motors can easily switch to generator mode and transmit energy back to the battery. In terms of applying such motors in quadcopters, they compete with brushless DC motors, but are much cheaper. Besides, their control circuits are also much cheaper and simpler.

Hundreds of different PM DC micromotors with different speeds of 4500 - 63000 rpm and supply voltage of 3-24 V PS are now available in online stores. Modern developments in the field of microprocessor technology (Arduino and others) and power electronics (PWM devices) have considerably facilitated their application, expanded the control of such motors, and created preconditions for the development of new controlled micro-electric drives and MEMS devices. This field of research is of vital importance because PM DC micromotors have significant active anchor resistance and soft mechanical characteristics respectively, as proven by experimental studies. That is, when the load is applied, their speed decreases significantly. Therefore, in order to be applied in micro drives and microactuators, both feedback and a controller are required to stabilize the rotation speed when the load changes.

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Another micromotor feature is a relatively low rotor inertia moment. As a result, electromechanical time constant T_m becomes commensurate with the electric time constant T_a of the armature, which may lead to overshooting of the output speed coordinate and limits some approaches to simplifying transfer functions (TF) of such motors [2].

The main purpose of this study is to investigate the possibility of approximating complex speed transfer function of PM DC micromotor by more compact fractional order transfer function and to estimate approximation accuracy. Such studies are not possible without the development of a high-precision model of PM DC micromotor by means of the parameters obtained through a series of experimental research.

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I. PM DC MICROMOTOR MODEL

To begin with, we consider the traditional PM DC micromotor electrical equivalent circuit [1,2,3,4] shown in Fig.1, where: u - DC voltage, $e(t)$ - back EMF, $i(t)$ - current, $T_e(t)$ - electromagnetic torque, $T_l(t)$ - load torque, $rn(t)$ - speed, J- PM DC inertia moment.

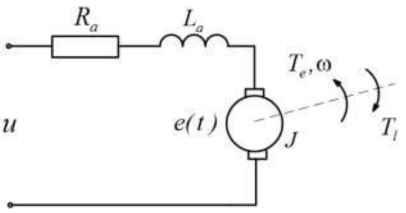

Fig. 1. PM DC micromotor electrical equivalent circuit

If we move from the electrical circuit to the model of the PM DC micromotor in the form of TF, we obtain the following model in the form of a block diagram shown in Fig.2, where $k_t = k_a$ ®$_5$ - torque constant, $k_e = k_a$ ®$_5$ - back EMF constant, k_a - armature constant, R_a - armatute circuit resistance, L_a - armature circuit inductance, $T_e = k_a l_a$ ®$_5 = k_t I_a$ - electromagnetic torque, $e = k_a$ra®$_5$ = = k$_e$ra - back EMF expression.

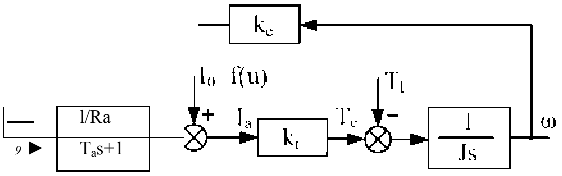

Fig. 2. PM DC micromotor block diagram

Based on block diagram in Fig. 2, the equivalent transfer function of motor speed is as follows:

$$WJs) = \frac{k_t}{JR_aT_as + JR_as + k_ek_t} \qquad (1)$$

where $T_a = L_a/R_a$ - the electric time constant of the motor.

If we perform the transformation of TF (1), given that the electromechanical time constant $T_m = JR_a/k_ek_t$, then we get the resulting expression:

$$WJs) =- \frac{1\ /ke}{T_aT_mS + T_ms + 1} \qquad (2)$$

TF (2) does not take into account B (rotor dumping) as an important parameter. In order to investigate the block diagram shown in Fig. 2, which corresponds to the TF (2), we must obtain the dependence between PM DC micromotor no-load current (I$_0$) and supply voltage (u) $I_0 = f(u)$.

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Another approach to modelling the mechanical part of PM DC micromotor is by means of using the B parameter. The TF of this mechanical part is expressed as follows [1]:

$$Wm(s) = \frac{kt}{Js + B}$$

In this case, the dependence $I_0 = f(u)$ may not necessarily be applied, but determination of the B parameter by experimental studies is a must. The equivalent TF of the loop shown in Fig. 2 is expressed as follows:

$$WJs) = \frac{k_t}{JR_aT_aS + (JR_a + BR_aT_a)s + BR_a + k_ek_t} \qquad ☐ \ (3)$$

If we perform the transformation of TF (3) provided the electromechanical time constant $T_m = JR_a/k_ek_t$, we obtain

$$WJs) = \frac{1\ /ke}{TaTms^2 + (Tm + \frac{T}{J})s + {}^a \quad +1} \qquad (4)$$

The obtained results (2) and (4) have demonstrated that the equivalent TF of PM DC micromotor speed is quite complex. Synthesis of a controller on its basis provides for the following options: either we need to synthesize this controller by some intelligent methods (PSO, genetic algorithm, etc.), which is quite difficult and time consuming, or we may traditionally use a practical approach where it is believed that the TF speed component $T_aT_ms^2$ is very low- impact and can be neglected T_aT_m «0.

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$$W(s) = -\frac{k}{a,s^1 + 1} \qquad (5)$$

Approximation of the PM DC micromotor speed TF by a more compact fractional order transfer function will be performed by means of using the intelligent particle swarm optimization method [6,7].

III. EXPERIMENTAL STUDIES OF PM DC MICROMOTOR

In this research experimental studies are conducted via the example of a widely used 365 series of micromotors. This series provides for a large number of various options with different rated parameters (power, supply voltage, speed) for the same motor size.

The first experimental study was conducted in a no-load mode by changing the motor supply voltage u and determining its current I_0 and speed n, which correspond to the set supply voltage. The results are shown in Table 1.

TABLE I. EXPERIMENTAL NO-LOAD MODE

№	No-load mode		
	u, V	*Io, mA*	*n, rpm*
1	12	41	2385
2	11	40	2182
3	10	39	1900
4	9	38	1687
5	8	37	1415
6	7	36	1176
7	6	35	912
8	5	34	648
9	4	33.5	400
10	3.5	32	0

The results of the study allowed to determine very important parameters for further model parameterization - no-load current I_0 = 41mA, no-load speed n_0= 2385 rpm at a rated supply voltage of 12 V. Another important parameter determined is the power consumption of the micromotor in no-load mode $P_{I0} = u \cdot I_a$ = 12 • 0.041* 0.5W.

Given that the efficiency factor of such micromotors is about 50%, we determine that the motor rated power is *1W .

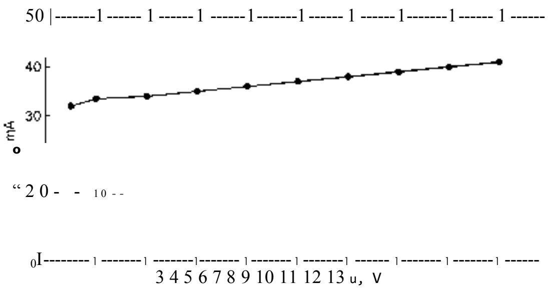

Fig. 3. Experimental diagram $I0 = f(u)$

The next experimental study was carried out in the mode of determining the motor operating characteristics. We determined the rated supply voltage of u = 12 V, started the motor and created a mechanical load on the motor shaft with the help of a mechanical loading device. During the experiment, we also determined its current I and rotation speed n .

The results of the abovementioned study are necessary for the

verification of the model accuracy, shown in Fig.2.

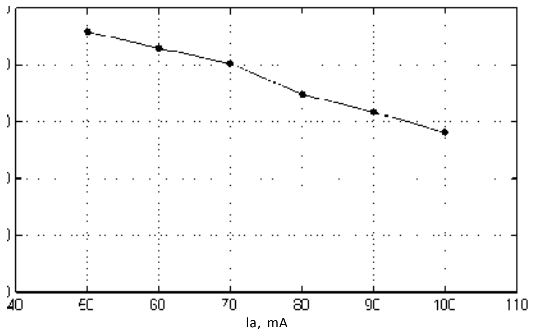

Fig. 4. Experimental diagram $n = f(I)$

Another experimental study was conducted to determine active resistance of the armature circuit. For this purpose, the motor rotor was brought to a halt, and the motor supply voltage creating a rated current in the armature circuit I_a = 100mA was set. The experiment was repeated ten times, and the average value of the applied voltage u = 6.745V was determined. Consequently, active resistance of the armature circuit was calculated as follows:

$$R_a = {}^{6745} = 67.45\text{Q} \,.\, {}^{a}\, 0.1$$

In this case, the resistance of the entire armature circuit is obtained, which includes resistance of the armature, commutator, brushes and connecting conductors. For further research we accept the assumption that the resistance of the commutator, brushes and connecting conductors is much lower than the resistance of the armature.

We supplied power and took the oscillogram of the voltage change process at the terminals of the PM DC micromotor (Fig. 5).

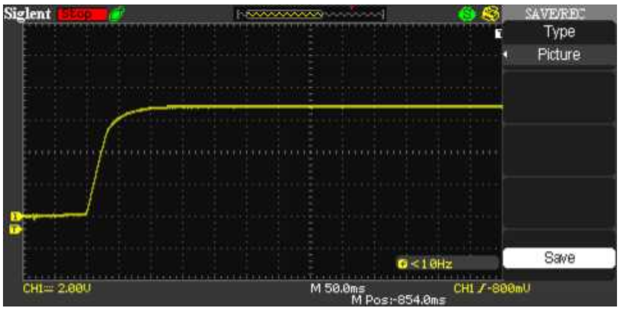

Fig. 5. Oscillogram of the voltage change process

Taking into account the equation describing the electric circuit of the motor

$$u = e(t) + L_a \frac{\sim TT\sim}{dt} + R_a i(t)\,' \tag{6}$$

in case the rotor is brought to a halt e = k_e— = 0 and the applied voltage u = 6.745B, equation (6) turns into

$$u = L_a \frac{di(t)}{dt} + R_a i(t), \tag{7}$$

The research has shown that the duration of the voltage change transition process at the terminals of a halted motor fully corresponds to the duration of the transition process of armature current I. Accordingly, unknown parameter L_a was established by Simulink modelling of the transition process of current (Fig. 2) for the duration of the transition process T=120ms. The results have shown that L_a = 1.248H.

researched motor, we determined:

$$k_e = \frac{8.52}{\text{ra}\quad 147.027} = 0.03411 \text{ Vs/rad.}$$

With regard to one of the assumptions in various models of PM DC micromotors in MATLAB [4], we consider that $k_t = k_e$ = 0.03411 Vs/rad. Inertia J = 0.5 • 10~^{6} kgm^2.

IV. APPROXIMATION OF INTEGER ORDER TF BY FRACTIONAL ORDER TF

Having all the necessary parameters, now let us check the initially made assumptions. Electric armature time constant T_a of PM DC micromotor has been calculated as

$$W—(s) =■ \frac{1\,/\,ke}{T_a T_m s + T_m s + 1} \qquad \frac{29.317}{0.0004885s^2 + 0.026s + 1} \tag*{.(8)}$$

When a step signal is applied to the input, the transition process of speed corresponding to the TF (8) is shown in Fig.6 (curve "1"). The obtained result has fully confirmed the assumption about the existence of overshooting and the impossibility of simplifying TF (2) by discarding the component $T_a T_m s^2$, because a completely different aperiodic process will be obtained (curve "3").

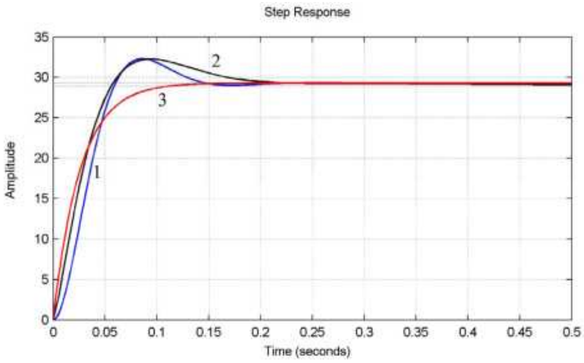

Fig. 6. TP of speed: "1" - TF (8), "2"-approximating TF (9), "3"-simlpified

$$W(s) = \frac{28.911}{0.0109s^{1.267} + 1} \tag{9}$$

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Fig. 7 shows respective Bode diagrams for TF (8) - "1" and for the approximating FO TF (9) - "2". The results of the transition processes comparison are shown in Table 3. Hence, FO TF provides much better stability.

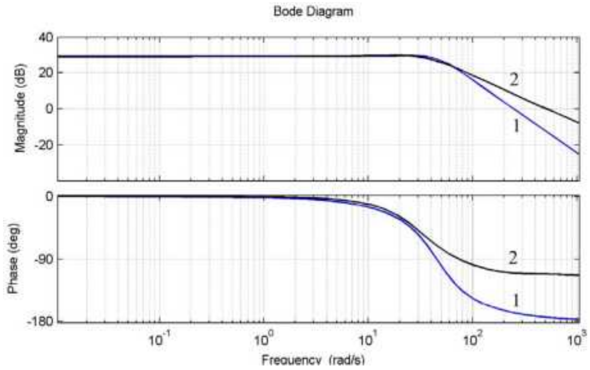

Fig. 7. Bode diagrams: curve "1"- TF (8), curve "2" - approximated TF (9)

Analysis of the accuracy of the transition and frequency approximation characteristics is based on the following parameters: t_{095} - time of transition process reaching the value of 0,95 of default value; ra_c - the crossover frequency; ^ - phase margin; ct - absolute standard deviation [5], 8 - relative approximation error [5].

TABLE II. APPROXIMATION ACCURACY

TF	$^t0.95$, ^{s}	, *rad/s*	^ , *deg*	CT	8 ,%
(8)	0,055	247.2	12.8	-	-
(9)	0,0505	513.3	68.93	0,9146	3,11

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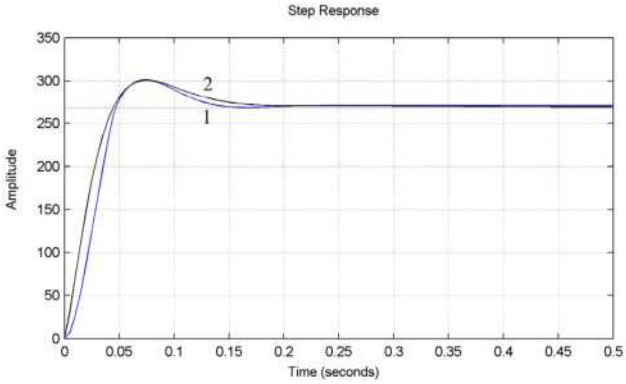

Fig. 8. TP of speed: curve "1" - model, curve "2" - approximating TF (9)

The FO TF (5) parameters in this case were determined by the PSO method and the following TF was obtained:

$$W(s) = \frac{22.2898}{0. \quad 0078s^{12817}+1} \tag{10}$$

The transition process of speed of the obtained fractional order TF (10) is shown in Fig.8 (curve "2"), with approximation error 8 =5.07%.

V. CONCLUSIONS

1. Different variants of obtaining TF of speed for PM DC micromotor modelling and the peculiarities of transfer functions simplification procedure for such motors have been considered. It is established that for this class of micromotors the electromechanical time constant T_m becomes commensurate with the electric time constant Ta of the armature, which leads to overshooting of the output coordinate, so simplification of TF of speed by discarding the s^2 component will result in aperiodic process and significant accuracy loss.

2. A series of experimental PM DC micromotor studies using a high-precision laser tachometer to obtain the necessary parameters for the high-accuracy micromotor model development have been carried out. The developed high-accuracy model made it possible to obtain the appropriate transition process of speed with overshooting and to draw conclusions about impossibility of neglecting of TF speed component $TaTmS^2$.

3. Approximation of complex transfer function of PM DC micromotor speed by a more compact fractional transfer function using the intelligent particle swarm optimization method has been performed. High accuracy of approximation has been estimated through the analysis of transition and frequency characteristics.

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[2]