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A MATHEMATICAL MODEL FOR DETERMINING AND ANALYZING TEMPERATURE REGIMES IN A BATTERY PACK OF ELECTRIC TRUCKS

A mathematical model for the determination of the temperature field and the analysis of temperature regimes in lithium-ion batteries have been developed. Using the theory of generalized functions, the thermophysical parameters of the structural components of a battery are represented by a single mathematical relation. A function in the form of the product of the generalized thermal conductivity coefficient for temperature was introduced, which avoided the differentiation of the product of two generalized functions as a result of constructing the initial differential equation of thermal conductivity, which was obtained with discontinuous coefficients. An analytical solution of this equation is determined, which is expressed by the temperature value at the conjugation surfaces of the layers of the structure. A relation was obtained to determine these values and expressions for constant integration. To determine the numerical values of the temperature in the design of the battery nodes, as well as to analyze the temperature gradients in its environment caused by the heterogeneity of the components due to heating, an algorithm and computational programs have been developed that allow to analyze lithium-ion batteries for their normal functioning. Using numerical programs, numerical values of the temperature were obtained for given values of the power of the internal heat sources, which made it possible to construct curves that reflect the behavior of the temperature field depending on the spatial coordinate. The angular points on the curve are revealed, which indicate the presence of a phase transition in the design of lithium-ion battery assemblies. As a consequence, it becomes possible to determine the permissible temperature values for the fire safety of these batteries.

Keywords: lithium-ion batteries; heat exchange; temperature field; temperature modes layered structures.

Introduction. The development of modern technologies and innovative developments from year to year make it possible for mankind to bring to life the most promising ideas, which even some years ago were considered unattainable. A prime example of this is the automotive industry, in particular the rapid reorientation of the internal combustion engine (ICE) market for electric vehicles. Electric cars are rapidly advancing in the markets of Europe and the United States, the number of which is more than 2 million units worldwide and is growing. Currently, the largest number of electric vehicles is in China, accounting for 32 % of the world's fleet of electric vehicles. The number of advanced economies has been steadily increasing since the abandonment of ICE cars in favor of electric trucks. Countries such as Norway, the Netherlands, Germany have already approved development programs at the state level, which foresee a complete abandonment of the DCE by 2030-2040.

The use of electric trucks in the field of human activity is effective in both economic and environmental and practical aspects, but the issue of fire safety of this mode of transport remains relevant. The main energy source in electric trucks is a rechargeable lithium-ion battery. Recent studies on the fire hazard of lithium-ion batteries indicate that, under conditions of overloading (short-circuiting) the battery and reaching a critical temperature in the medium, which reaches and above, conditions for irreversible combustion occur, depending on the physical and geometric parameters of the structure As a result, a lithium-ion battery can produce 6.0–7.8 kW of energy. Therefore, it is important to determine the permissible temperatures that facilitate the normal operation of lithium-ion batteries. For this reason, the purpose of the work is to develop mathematical models for the determination of temperature fields in lithium-ion batteries, which allow to establish acceptable tem-

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perature modes of their normal operation. The object of study is the processes of heat exchange in lump-homogeneous media, and the subject of the study are linear and nonlinear mathematical models of the heat transfer process and methods for determining the analytical solutions of the corresponding boundary value problems of mathematical physics.

Analysis of recent research and publications. The definition of temperature regimes in both homogeneous and heterogeneous structures has attracted the attention of many researchers [0, 2, 4, 8, 12, 14]. In [3, 7, 6], the existing and new approaches to the creation of mathematical models for the analysis of heat exchange between piecewise homogeneous structures and the environment and methods for solving linear and nonlinear boundary value problems for piecewise homogeneous environments are improved. Two- and three-dimensional models containing equations whose coefficients are functions of thermophysical properties of phases and geometric structure are considered. The methods of determination of analytical and analytical-numerical solutions of boundary value problems of thermal conductivity are given. Heat transfer processes in homogeneous and layered structures with alien inclusions of canonical form are investigated and analyzed. A mathematical model for the study of the temperature field caused by the rotational welding of metals by friction is developed. For this purpose, an axisymmetric nonlinear boundary-value thermal conductivity problem for two circular cylinders was formed with simultaneous consideration of thermal formation due to friction and from plastic deformation. It is accepted that the coefficient of friction, the plasticity limit and the thermophysical properties of the sample materials change with increasing temperature. The numerical solution of the problem is obtained by the finite element method. The calculation was performed for two samples made of AISI 1040 steel. It was found that friction heat formation has a greater influence on the maximum contact surface temperature compared to heating from plastic deformation. The empirical dependences of temperature on the parameter characterizing the share of each of the two specified mechanisms of heat formation in the total amount of heat generated are established. The design model was verified by comparing the calculated temperature values with the corresponding experimental ones [11]. A mathematical model was developed to determine the non-stationary temperature field in a multilayer heat-sensitive plate under different heating conditions. Applied to the formulated nonlinear boundary problem of Goodman transform and obtained by the method of straight lines using the integro-interpolation method, its semi-discrete analogue in the form of a Cauchy problem for a system of nonlinear ordinary differential equations. The Cauchy problem for a system of nonlinear ordinary differential equations in a two-layer plate is solved numerically using the backward differentiation formulas, specifying the temperature dependences for the thermophysical parameters of the plate layers. The temperature increase is determined by the Goodman variable by inverse interpolation formulas [5].

The object of study and its mathematical model. We describe the lithium-ion battery geometrically with a layered structure with evenly distributed internal heat sources whose power is equal $q_0 = const$. The layers differ in geometric (width) and thermophysical (coefficient of thermal conductivity) parameters. The given design is attributed to the Cartesian rectangular coordinate system, on whose surfaces $x = 0, x = x_n$ a constant value of temperature is given t_c. There is an ideal thermal contact on the surfaces $x = x_i\,(i = \overline{1, n-1})$ of the layers $t_i = t_{i+1},\ \lambda_i \frac{dt_i}{dx} = \lambda_{i+1}\frac{dt_{i+1}}{dx}$ (Fig. 1). In the above structure, it is necessary to determine the temperature distribution by the spatial coordinate, which we obtain by solving the heat conduction equation [9, 13]

$$\frac{d}{dx}[\lambda(x)\frac{dt}{dx}] = -q_0 \qquad (1)$$

with boundary conditions

$$t(0) = t(x_n) = t_c, \qquad (2)$$

where $$\lambda(x) = \lambda_1 + \sum_{i=1}^{n-1}(\lambda_{i+1} - \lambda_i)S_+(x - x_i) - \qquad (3)$$

coefficient of thermal conductivity of a non-uniform plate; λ_i – coefficient of thermal conductivity of materials of the i-th layer of structure; t_c – ambient temperature; $S_+(\varsigma)$ – asymmetric unit function; $S_+(\zeta) = \begin{cases} 1, & \zeta \geq 0, \\ 0, & \zeta < 0. \end{cases}$ [10].

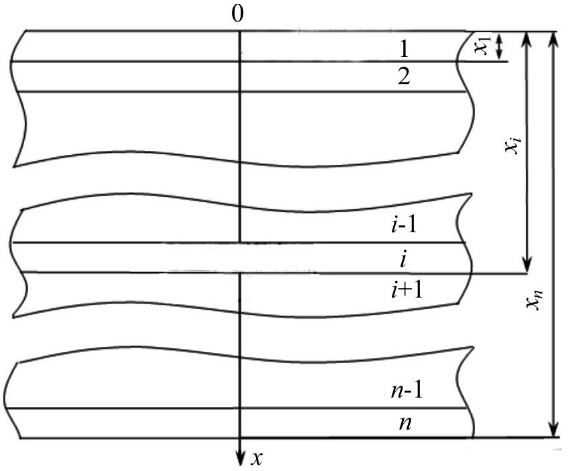

Figure 1. Geometric representation of the lithium-ion battery assembly

We introduce the function

$$T(x) = \lambda(x)t(x) \qquad (4)$$

and differentiate it by the variable, given expression (3) for the coefficient of thermal conductivity. The result is a ratio

$$\lambda(x)\frac{dt}{dx} = \frac{dT}{dx} - \sum_{i=1}^{n-1}\left(\lambda_{i+1} - \lambda_i\right)t(x_i)\delta_+(x - x_i),$$

taking into account the initial equation (1) is written in the form

$$\frac{d^2T}{dx^2} - \sum_{i=1}^{n-1}\left(\lambda_{i+1} - \lambda_i\right)t(x_i)\delta'_+(x - x_i) = -q_0.$$

The general solution of this equation is defined by the expression

$$T(x) = \sum_{i=1}^{n-1}(\lambda_{i+1} - \lambda_i)t(x_i)\,S_+(x - x_i) - \frac{q_0}{2}x^2 + c_1x + c_2, \qquad (5)$$

where c_1, c_2 are the steel integrations.

We find $t(x_i)\,(i = \overline{1, n-1})$ the quantities using relation (5) as

$$t(x_1) = \frac{1}{\lambda_1}\left(c_1x_1 + c_2 - \frac{q_0}{2}x_1^2\right),$$

$$t(x_i) = c_1\left[\sum_{j=1}^{i-1}\Lambda_j x_j + \frac{x_i}{\lambda_i}\right] + \frac{c_2}{\lambda_1} - \frac{q_0}{2}\left[\sum_{j=1}^{i-1}\Lambda_j x_j^2 + \frac{x_i^2}{\lambda_i}\right],$$

where $\Lambda_j = \frac{1}{\lambda_j} - \frac{1}{\lambda_{j+1}}$.

Considering the boundary conditions (2) and formula (5), we define the integration steels c_1, c_2. As a result, we get the following expressions:

$$c_1 = 0,5q_0 \frac{\lambda_n \sum\limits_{i=1}^{n-1} \Lambda_j x_i^2 + x_n^2}{\lambda_n \sum\limits_{i=1}^{n-1} \Lambda_j x_i + x_n}; c_2 = \lambda_1 t_c.$$

Therefore, the temperature field in the structures of the nodes of the layered structure of lithium-ion batteries is completely determined by formula (5).

Analysis of numerical results. In Fig. 2 shows the behaviour of the temperature field in the construction of a five-layer lithium-ion battery assembly in which the material of the first, third and fifth layers is aluminium ($\lambda_1 = \lambda_3 = \lambda_5 = 282$ W/(Km)) (at a temperature of 627 °C) and the second and fourth ones are lithium ($\lambda_2 = \lambda_4 = 52,9$ W/(Km)) (at a temperature of 627 °C) for such thickness values layers: $x_1 = x_3 - x_2 = x_5 - x_4 = 0,05$ m; $x_2 - x_1 = x_4 - x_3 = 0,20$ m.

As can be seen from the figure of the highest value, the temperature reaches the average aluminium layer and decreases monotonically as a function of the spatial coordinate to a given value in boundary conditions (2). The angular points observed on curves in the inner surfaces of the first and fifth aluminium layers of the lithium battery assembly indicate that the temperature is continuous as a function of the spatial coordinate and that at these points is a phase transition from the medium to the aluminium (solid state) into the lithium medium (liquid state).

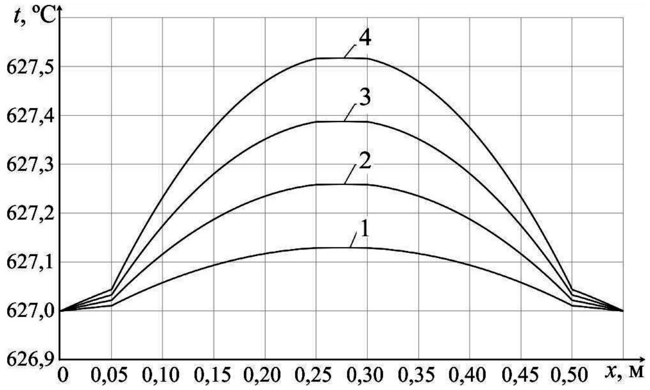

Figure 2. Temperature distribution in the design of a lithium-ion battery assembly for different power q_0 source values: 1 – q_0 = 250 W/m^3; 2 – q_0 = 500 W/m^3; 3 – q_0 = 750 W/m^3; 4 – q_0 = 1000 W/m^3

Conclusions. A mathematical model for the determination of the temperature field and the analysis of temperature modes in lithium-ion batteries have been developed, the construction of which is represented by a layered structure. The theory of generalized functions was applied, which made it possible to conveniently describe the thermophysical parameters of the design of a battery node of a piecewise homogeneous structure. A new function was introduced in the form of the product of a generalized thermal conductivity coefficient for temperature, which allowed to obtain the initial differential equation of thermal conductivity with discontinuous coefficients. An analytical solution of the boundary value problem is obtained, which is expressed by the temperature value on the surfaces of the conjugation of the layers of the structure. Analytical ratios were obtained to determine these values. An algorithm and software for the analysis of temperature regimes in the design of nodes of lithium-ion batteries of layered structure have been developed, which allow to determine the permissible values of temperature for their normal functioning. The angular points on the curve of behavior of the temperature field for the specified values of the power of the internal heat sources are revealed, which indicate the presence of a phase transition in the design of the lithium-ion battery units.

References

1. Bayat, A., Moosavi, H., & Bayat, Y. (2015). Thermo-mechanical analysis of functionally graded thick spheres with linearly time-dependent temperature. *Scientia Iranica, 22*(5), 1801–1812.
2. Carpinteri, A., & Paggi, M. (2008). Thermoelastic mismatch in nonhomogeneous beams. *J. Eng. Math., 61*(2–4), 371–384.
3. Gavrysh, V. I., & Fedasjuk, D. V. (2012). *Modeljuvannja temperaturnyh rezhymiv u kuskovo-odnoridnyh strukturah*. Lviv: Publishing house of Lviv Polytechnic National University, pp. 176–178. [In Ukrainian].
4. Ghannad, M., & Yaghoobi, M. P. (2015). A thermoelasticity solution for thick cylinders subjected to thermo-mechanical loads under various boundary conditions. *Int. Journal of Advanced Design & Manufacturing Technology, 8*(4), 1–12.
5. Harmatii, H. Yu., Popovych, V. S., & Krul, M. M. (2019). Vplyv termochutlyvosti materialu na neustalenyi teplovyi stan bahatosharovoi plastyny. *Fizyko-khimichna mekhanika materialiv, 1,* 98–104. [In Ukrainian].
6. Havrysh, V. I., Kolyasa, L. I., & Ukhanka, O. M. (2019). Determination of temperature field in thermally sensitive layered medium with inclusions. *Scientific Bulletin of NHU, 1,* 94–100.
7. Havrysh, V. I., Baranetskiy, Ya. O., & Kolyasa, L. I. (2018). Investigation of temperature modes in thermosensitive non-uniform elements of radioelectronic devices. *Radio Electronics, Computer Science, Control, 3*(46), 7–15.
8. Jabbari, M., Karampour, S., & Eslami, M. R. (2011). Radially symmetric steady state thermal and mechanical stresses of a poro FGM hollow sphere. *International Scholarly Research Network ISRN Mechanical Engineering, 3,* 1–7. https://doi.org/10.5402/2011/305402
9. Koliano, Iu. M. (1992). *Metody teploprovodnosti i termouprugosti neodnorodnogo tela*. Kyiv: Scientific thought, 280 p. [In Russian].
10. Korn, G., & Korn, T. (1977) *Spravochnik po matematike dlia nauchnykh rabotnikov i inzhenerov*. Moscow: Science, 640 p. [In Russian].
11. Lukashevych, A. (2019). Temperaturne pole u zoni kontaktu pid chas rotatsiinoho zvariuvannia metaliv tertiam. *Fizyko-khimichna mekhanika materialiv, 1,* 41–46. [In Ukrainian].
12. Mohazzab, A. H., & Jabbari, M. (2011). Two-Dimensional Stresses in a Hollow FG Sphere with Heat Source. *Advanced Materials Research,* 264–265, 700–705. https://doi.org/10.4028/scientific.net/amr.264-265.700
13. Podstrigach, Ia. S., Lomakin, V. A., & Koliano, Iu. M. (1984). *Termouprugost tel neodnorodnoi struktury*. Moscow: Science, 350 p. [In Russian].
14. Yangian, Xu., & Daihui, Tu. (2009). Analysis of steady thermal stress in a ZrO2/FGM/Ti-6Al-4V composite ECBF plate with temperature-dependent material properties by NFEM, *WASE. Int. Conf. on Informa. Eng.*, 2–2, 433–436.

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МАТЕМАТИЧНА МОДЕЛЬ ВИЗНАЧЕННЯ ТА АНАЛІЗУ ТЕМПЕРАТУРНИХ РЕЖИМІВ У ПАКЕТІ АКУМУЛЯТОРНОЇ БАТАРЕЇ ЕЛЕКТРОКАРІВ

Розроблено математичну модель визначення температурного поля та аналізу температурних режимів у літій-іонних батареях, конструкцію вузлів яких геометрично описано шаруватою структурою. Із використанням теорії узагальнених функцій теплофізичні параметри конструкційних складових вузлів батареї зображено єдиним математичним співвідношенням. Запроваджено функцію у вигляді добутку узагальненого коефіцієнта теплопровідності на температуру, що дало змогу уникнути диференціювання добутку двох узагальнених функцій внаслідок побудови вихідного диференціального рівняння теплопровідності, яке отримано з розривними коефіцієнтами. Визначено аналітичний розв'язок цього рівняння, який виражений через значення температури на поверхнях спряження шарів структури. Отримано співвідношення для визначення цих значень та вирази щодо сталих інтегрування. Для визначення числових значень температури в конструкції вузлів батареї, а також аналізу температурних градієнтів в її середовищі, зумовлених неоднорідністю складових внаслідок нагрівання, розроблено алгоритм та обчислювальні програми, які дають змогу аналізувати літій-іонні батареї щодо їх нормального функціонування. Із використанням обчислювальних програм отримано числові значення температури для заданих значень потужності внутрішніх джерел нагрівання, що дало змогу побудувати криві, які відображають поведінку температурного поля залежно від просторової координати. Виявлено кутові точки на кривій, які свідчать про наявність фазового переходу в конструкції вузлів літій-іонних батарей. Як наслідок, стає можливим визначити допустимі значення температури щодо пожежної безпеки цих батарей.

Ключові слова: літій-іонні батареї; теплообмін; температурне поле; температурні режими; шаруваті структури.