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PATH-DEPENDENT INFINITE-DIMENSIONAL SDE: AN ENTROPY APPROACH

S. Rœlly

We present some recent existence (and uniqueness) results on weak solutions of infinite-dimensional stochastic differential equations driven by a Brownian term. The drift function is very general, in the sense that it is path-dependent and non-regular. The originality of our method leads in the use of the specific entropy as a tightness tool and on a description of such stochastic differential equation as solution of a variational problem on the path space.

The talk is based on joint works with P. Dai Pra and D. Dereudre.

References

[1] Dai Pra, P., Roelly, S., *An existence result for infinite-dimensional Brownian diffusions with non-regular and non Markovian drift.* Markov Processes and Related Fields **10**, 2004, 113–136.

[2] Dereudre, D., Roelly, S., *Path-dependent infinite-dimensional SDE with non-regular drift: an existence result.* Preprint, 2014.

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CONTINUOUS DEPENDENCE OF SOLUTIONS TO SDE'S DRIVEN BY FRACTIONAL BROWNIAN MOTION ON THE HURST INDEX OF A DRIVING SIGNAL

T. Shalaiko

We consider the following d-dimensional stochastic differential equation:

$$dX_t^H = a(X_t^H)dt + b(X_t^H)dB_t^H, \tag{1}$$

where $a\colon \mathbb{R}^d \to \mathbb{R}^d$, $\sigma\colon \mathbb{R}^d \to \mathbb{R}^{d\times m}$ and $B^H = \{B_t^H, t \in \mathbb{R}\}$ is an m-dimensional fractional Brownian motion with Hurst index $H \in (0,1)$. Results from the rough path analysis imply that the law of X^H depends continuously on H, e.g. $\mathsf{P}_{X^H} \to \mathsf{P}_{X^{H_0}}$, when $H \to H_0$ in case of $H \geq 1/2$.

We are especially interested in the additive noise case ($b = 1$) when a additionally satisfies non-dissipative condition:

$$\langle a(x) - a(y), x - y\rangle \leq -L|x-y|^2, \quad \text{for a non-negative constant } L.$$

In this case it is known [1] that for any H exists the initial condition $x_0^H \in \mathbb{R}^d$, such that the solution X_t^H of (1) with $X_0^H = x_0^H$ is stationary.

If a Gaussian field $B = \{B_t^H, H \in (0,1), t \in \mathbb{R}\}$ is given by a Mandelbrot-van Ness representation we establish a pathwise continuous dependence for solutions, with the same initial values, on $H \in (0,1)$ and the similar result for the stationary solutions (in this case initial values are not necessary the same). We also discuss some applications to the dynamical systems. This is joint work with M. J. Garrido-Atienza, P. Kloeden and A. Neuenkirch.

References

[1] Garrido-Atienza, M. J., Kloeden, P., Neuenkirch, A. *Discretization of stationary solutions of stochastic systems driven by fractional Brownian motion.* Apply. Math. Optim. **60**, 2009, 151–172.

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STOCHASTIC APPROXIMATION PROCEDURE WITH INDEPENDENT INCREASE IN LOCAL BALANCE CONDITION

O. I. Shvets[1], Ya. M. Chabanyuk[2]

The stochastic approximation procedure with impulse perturbation in an ergodic Markov environment in diffusion approximation schema is defined by the stochastic differential equation [1]:

$$du^\varepsilon(t) = a(t)[C(u^\varepsilon(t); x(\frac{t}{\varepsilon^2}))dt + d\eta^\varepsilon(t)], u^\varepsilon(0) = u_0,$$

where $C(u;x), u \in R^d$ is a regression function, $u^\varepsilon(t), t \geq 0$ is a random evolution, and ε is a small series parameter.

The impulse perturbation process $\eta^\varepsilon(t) := \varepsilon\eta(\frac{t}{\varepsilon^2}), t \geq 0$ is defined by generators:

$$\Gamma(x)\varphi(w) = \int_{R^d} [\varphi(u+v) - \varphi(u)]\Gamma(u; dv; x), u \in R^d, x \in X.$$

Hence, the process $\eta^\varepsilon(t), t \geq 0$ on the test-functions $\varphi(u) \in C^3(R^d)$ is defined by generator:

$$\Gamma^\varepsilon(x)\varphi(u) = \varepsilon^{-2}\int_{R^d} [\varphi(u+\varepsilon v) - \varphi(u)]\Gamma(u; dv; x).$$

Theorem 1. *Let there exists the Lyapunov-function $V(u) \in C^3(R^d)$, for the averaged dynamic system $du(t) = C(u(t))dt$, which satisfy the following conditions:*

$C1: C(u)V'(u) < -cV(u), c > 0,$

$C2: |B(x)V(u)| \le c_1(1 + V(u)), c_1 > 0,$

$C3: |\delta_\Gamma^\varepsilon(u;x)V(u)| \le c_2(1 + V(u)), c_2 > 0,$

$C4: |\boldsymbol{C}(x)\boldsymbol{R}_0\tilde{\boldsymbol{C}}(x)V(u)| \le c_3(1 + V(u)), c_3 > 0,$

$C5: |\boldsymbol{B}(x)\boldsymbol{R}_0\tilde{\boldsymbol{C}}(x)V(u)| \le c_4(1 + V(u)), c_4 > 0,$

$C6: |\delta_\Gamma^\varepsilon(u;x)\boldsymbol{R}_0\tilde{\boldsymbol{C}}(x)V(u)| \le c_5(1 + V(u)), c_5 > 0,$

where $\tilde{\boldsymbol{C}}(x)V(u) = [\boldsymbol{C}(x) - \boldsymbol{L}]V(u)$, $\boldsymbol{L}V(u) = \Pi C(u;x)V'(u)$, $\boldsymbol{B}(x)V(u) = B(u;x)V''(u)$, *and* $\|\delta_\Gamma^\varepsilon(u;x)V(u)\| \to 0$, *while* $\varepsilon \to 0$. *Let the local balance condition holds:* $b(u;x) := \int_{R^d} v\Gamma(u;dv;x) \equiv 0$, *and the normalizing function* $a(t) > 0$ *satisfy the conditions* $\int_0^\infty a(t)dt = \infty$, $\int_0^\infty a^2(t)dt < \infty$. *Then the solution of the stochastic differential equation converges with the probability 1 to the equilibrium point* u^*, *which is defined by the equation* $C(u^*) = 0$.

References

[1] Kororlyuk, V. S., *Random evolutions with locally independent increments on increasing time intervals.* Journal of Mathematical Sciences **179** (2), 2011, 273–289.

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ON THE INCREASE RATE OF RANDOM FIELDS ON UNBOUNDED DOMAINS

A. I. Slyvka-Tylyshchak

The estimates for distribution of supremum for the normalized φ-sub-Gaussian random fields defined on the unbounded domain are found. In particular, we have obtained the estimates for distribution of supremum for the normalized solution of the hyperbolic equation of mathematical physics.

Theorem 1. *Let* $\{\xi(x,t),\ (x,t) \in V\}$, $V = [-A;A] \times [0,+\infty)$ *be a separable random field belonging to* $Sub_\varphi(\Omega)$. *Assume also that the following conditions are satisfied.*

(1) $[b_k, b_{k+1}]$, $k = 0,1,\dots$ *is a family of such segments, that*

$$0 \le b_k < b_{k+1} < +\infty, \quad k = 0,1,\dots \quad V_k = [-A;A] \times [b_k, b_{k+1}], \quad \bigcup_k V_k = V.$$

(2) There exist the increasing functions $\sigma_k(h)$, $0 < h < b_{k+1} - b_k$, *such that* $\sigma_k(h) \underset{h\to 0}{\longrightarrow} 0$,

$$\sup_{\substack{|x-x_1|\le h,\\ |t-t_1|\le h\\ (x,t),(x_1,t_1)\in V_k}} \tau_\varphi\left(\xi(x,t) - \xi(x_1,t_1)\right) \leqslant \sigma_k(h)$$

and $\int\limits_{0+} \Psi\left(\ln \frac{1}{\sigma_k^{(-1)}(\varepsilon)}\right) d\varepsilon < \infty$, *where* $\Psi(u) = \frac{u}{\varphi^{(-1)}(u)}$, $\sigma_k^{(-1)}(\varepsilon)$ *is an inverse function to* $\sigma_k(\varepsilon)$.

(3) $c = \{c(t),\ t \in R\}$ *is some continuous function, such that* $c(t) > 0$, $t \in R$, $c_k = \min\limits_{t\in[b_k,b_{k+1}]} c(t)$.

(4) $\sup\limits_k \frac{\varepsilon_k}{c_k} < \infty$, $\sup\limits_k \frac{I_\varphi(\theta\varepsilon_k)}{c_k} < \infty$.

(5) The series $\sum\limits_{k=0}^{\infty} \exp\left\{-\varphi^*\left(\frac{sc_k(1-\theta)}{2\varepsilon_k}\right)\right\}$ *converges for some* s *in such a way that* $\sup\limits_k \frac{4\varepsilon_k}{c_k(1-\theta)} < s < \frac{u}{2}$, *where* $\varepsilon_k = \sup\limits_{(x,t)\in V_k} \tau_\varphi(\xi(x,t))$, $k = 0,1,\dots$

Then

$$P\left\{\sup_{(x,t)\in V} \frac{|\xi(x,t)|}{c(t)} > u\right\} \leqslant 2\exp\left\{-\varphi^*\left(\frac{u}{s}\right)\right\} \cdot \sum_{k=0}^{\infty} \exp\left\{-\varphi^*\left(\frac{sc_k(1-\theta)}{2\varepsilon_k}\right)\right\} = 2A(u),$$

for $u > \sup\limits_k \frac{\tilde{I}_\varphi(\theta\varepsilon_k)}{c_k} \cdot \frac{4}{\theta(1-\theta)}$, *where* $0 < \theta < 1$, $\tilde{I}_\varphi(\delta) = \int\limits_0^\delta \Psi\left[\left(\ln\left(\frac{A}{\sigma_k^{(-1)}(\varepsilon)} + 1\right)\right) + \left(\ln\left(\frac{b_{k+1}-b_k}{2\sigma_k^{(-1)}(\varepsilon)} + 1\right)\right)\right] d\varepsilon$, $k = 0,1,\dots$

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