Increasing the efficiency of the application of discrete Walsh signals in telecommunication systems

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Abstract**—**The paper analyzes correlation functions of discrete Walsh signals obtained on the basis of Walsh functions. The advantage of these signals is the zero value of the main petal of the cross-correlation function (CCF), which indicates their orthogonality. But a significant drawback of these signals is a significant level of unwanted side lobes of both the CCF and the autocorrelation function (ACF). Based on the analysis of the features of CCF and ACF of discrete Walsh signals, a method of increasing the efficiency of their use in telecommunication systems (TCS) is proposed.

Keywords **—** Walsh functions and discrete signals, side petals of CCF and ACF, broadband TCS using discrete Walsh signals.

I. Introduction

Code sequences (codes) are widely used in various aspects of application (TCS). These include [1] groups of N rectangular pulses (chips) with the same duration τ and level +1 or -1. They are formed according to known dependencies, and their period T is

$$T = N\,\tau \qquad (1)$$

Thus, codes refer to periodic digital signals.

The specified code sequences also include discrete Walsh signals. They are formed on the basis of Walsh functions [2], and some of them (length 8) are shown in Fig. 1.

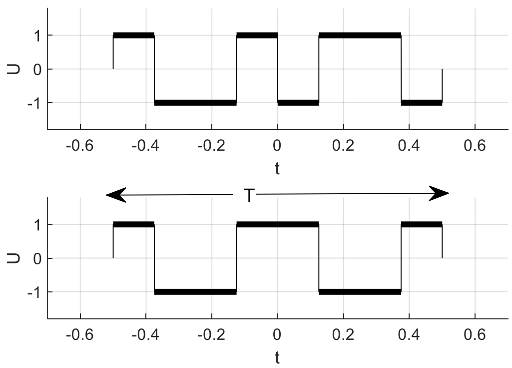

Fig.1.Walsh functions with period T

Walsh functions have one very important property - they are orthogonal, i.e. the value of the main lobe of the CCF is equal to 0. This property is unique and is present only in rather limited types of analytical functions, such as harmonic functions (sin, cos) and Barker functions. Due to the orthogonality of the Walsh function, it is widely used in the spectral analysis of signals.

In the process of further development and improvement of TCS, broadband signals are increasingly used. The basis for their formation are the code sequences mentioned above, and the main requirement for them is “good” characteristics of their correlation functions. The desired characteristics of CCF, AKF include: orthogonality, minimum value of side petals and minimum number of side petals with maximum levels.

In this regard, Walsh codes are of particular interest. They are based on the approximation of Walsh functions by appropriate codes that contain a certain number of chips. For example, based on the Walsh functions (Fig. 1), the following Walsh codes (Fig. 2) are formed.

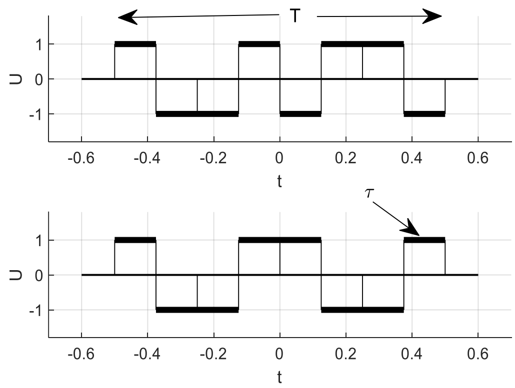

Fig. 2. Walsh codes with period T

The main difference between functions and Walsh codes is that the functions contain rectangular pulses of different durations, while the codes contain the same duration.

It is most convenient to generate Walsh codes using the Hadamard matrix. Each strip of this matrix contains information about one of the Walsh codes, and for clarity, the values of the chips "-1" and "+1" are marked "+" and "-", respectively. A code of length N=1 is determined based on the following Hadamard matrix

$$\mathbf{H_1} = + \qquad (2)$$

The Hadamard matrix H2 for codes (N=2) is formed on the basis of the matrix H1

$$\mathbf{H_2} = \begin{matrix} \mathbf{H_1} & \mathbf{H_1} \\ \mathbf{H_1} & \mathbf{-H_1} \end{matrix} = \begin{matrix} + & + \\ + & - \end{matrix} \qquad (3)$$

Similarly, the Hadamard H4 matrix for codes (N=4) is formed on the basis of the H2 matrix

$$\mathbf{H_4} = \begin{matrix} \mathbf{H_2} & \mathbf{H_2} \\ \mathbf{H_2} & \mathbf{-H_2} \end{matrix} = \begin{matrix} + & + & + & + \\ + & - & + & - \\ & & & \\ + & + & - & - \end{matrix} \qquad (4)$$

+ - - +

Figure 3 shows the Walsh codes obtained according to the H4 matrix

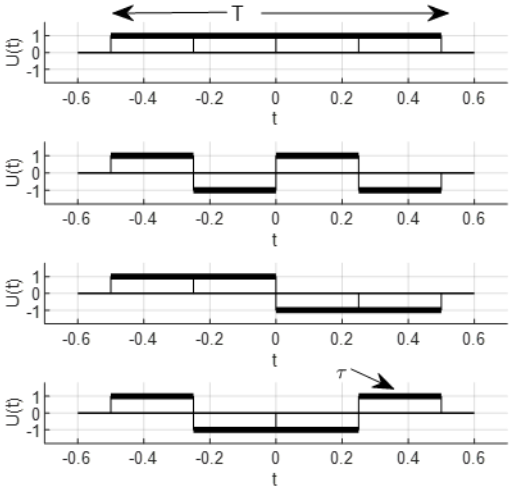

Fig. 3. Walsh codes (N=4)

When studying an array of Walsh codes, there is a need for their numbering. It can be seen that the resulting 4 codes change polarity a different number of times (from 0 to N-1). In the Walsh-Harmuth numbering system, the code number corresponds to the number of times the indicated polarity changes. Therefore, the specified numbering can be considered ordered by frequency.

The paper examines CCF and ACF using the example of Walsh codes with N=8 (similar studies can be conducted for Walsh codes of any length, which does not change the fundamental conclusions from the research results).

H₈ = H₄ H₄ = + + + +   + + + +

H₄ -H₄   + - + -   + - + -

+ + - -   + + - -

+ - - +   + - - + (5)

+ + + +   - - - -

+ - + -   - + - +

+ + - -   - - + +

+ - - +   - + + -

Figure 4 shows the Walsh codes obtained according to the H8 matrix and numbered according to the Walsh-Harmuth numbering

It can be seen that an increase in the code number indicates an increase in the frequency of its change.

Comparing the Hadamard matrices (2) ... (5) for Walsh codes of different lengths, we can conclude that they can simply be implemented using digital automata. This also refers to the essential advantages of Walsh codes [3].

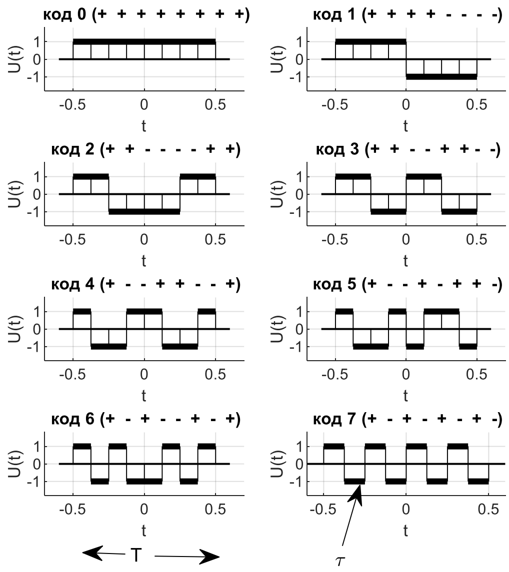

Fig. 4. Walsh codes (N=8)

When determining the correlation functions of codes, their normalized value (relative to the maximum) is used. Then the normalized VKF of two codes (u, v) of length N (that is, with the number of chips N) is determined as follows

$$r_{uv}(m) = 1/N \sum_{i=m+1}^{N} u_i\, v_{i-m}$$

$$r_{uv}(m) = 1/N \sum_{i=+1}^{N+m} u_i\, v_{i-m} \qquad (6)$$

It is obvious that when the following condition (u=v) is met, the ACF is determined based on dependence (6).

II. MAIN PETAL OF CORRELATION FUNCTIONS OF WALSH CODES

Table 1 shows the definition of the main petal of the CCF for two arbitrary Walsh codes, for example, codes 2 and 5 (Fig. 4). According to dependence (6), when determining the main petal, the codes are synchronized, that is, they start at the same time.

Table 1

i	1	2	3	4	5	6	7	8
u_i (code 2)	1	1	-1	-1	-1	-1	1	1
v_i (code 5)	1	-1	-1	1	-1	1	1	-1
$u_i\, v_i$	1	-1	1	-1	1	-1	1	-1
$\sum u_i\, v_i$	r_{uv} = 1*4+(-1)*4=0							

It can be seen that the main petal of the indicated codes is equal to zero, that is, they are orthogonal. Similarly, it can be shown that any other Walsh codes are also orthogonal [4]. It is obvious that when determining the correlation functions of a significant number of Walsh codes, especially codes of a significant length N, it is advisable to use software tools, in particular, the MATLAB.

Accepting the codes in table 1 as the same (for example, code 2), it is possible to similarly determine the value of the main petal of the ACF for this code (table 2).

Table 2

i	1	2	3	4	5	6	7	8
u_i (code 2)	1	1	-1	-1	-1	-1	1	1
u_i (code 2)	1	1	-1	-1	-1	-1	1	1
u_i v_i	1	1	1	1	1	1	1	1
r_{uu} (m = 0)	1/8*(1*8)=1							

It should be noted that the value 1 is the maximum for the normalized correlation function, both CCF and ACF, for any code (not only Walsh).

III. STUDY OF ACF OF WALSH CODES

The study of ACF of Walsh codes consists, first of all, in the determined levels of its side lobes. At the same time, the studied code and its copy, in contrast to the definition of the main petal, cease to be synchronized. In this case, the copy of the investigated code is shifted (lags or leads) relative to the code itself by one or more chips. Both periodic (the code contains one period T, and its copy - several periods) and aperiodic (the code and the copy contain one period) normalized ACF are considered. Next, only aperiodic normalized ACFs are considered. Their analysis makes it possible to obtain fairly complete information about the features of various Walsh codes.

For example, Table 3 shows code 2 and its copy, which is one chip later (m=1 in dependence (6)). As a result, the value of one of the side petals of the normalized ACF code 2 was obtained

Table 3

i	1	2	3	4	5	6	7	8	
u_i (код 2)	1	1	-1	-1	-1	-1	1	1	
u_{im}, m=1 (код 2)		1	1	-1	-1	-1	-1	1	1
u_i u_{im}	0	1	-1	1	1	1	-1	1	0
r(u_i, u_{im})	1/8*(1*5 +(-1)*2) =3/8 = 0.375								

It can be seen that the side petal in this case is quite significant and is 37.5% of the main petal of the ACF. As indicated above, an increase in the level of side lobes worsens the probability of an error in the received signal. Therefore, in the "ideal" code, side lobes should be absent at all.

Figure 5 shows information about the normalized ACF of Walsh, indicated in Figure 4.

On the basis of the obtained results, the following conclusions can be drawn regarding the normalized ACF of Walsh codes and offer suggestions regarding the simplification of their further analysis:

a) ACFs are symmetrical relative to the main petal, i.e. when "shifting" a copy of the code to the right (see Table 3), similar results will be obtained, as when shifting a copy to the left;

b) therefore it is suggested to use only part of the ACF (for example, when "shifting" the copy only to the right);

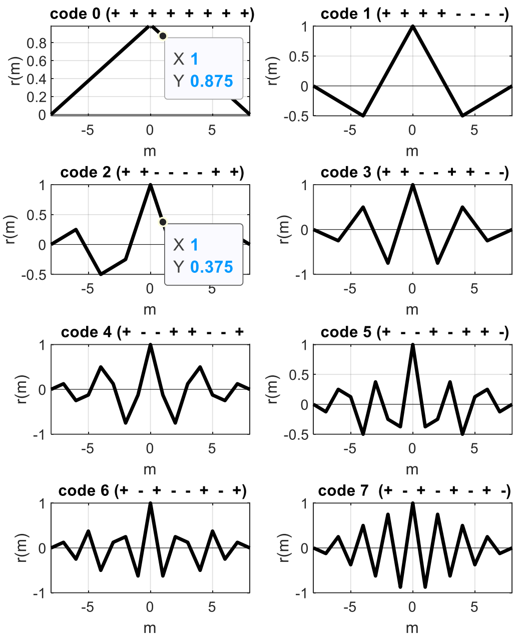

Fig. 5. ACF codes, according to the numbering of the Walsh-Harmuth system, (N=8)

c) the level of side petals is quite significant, and their largest value is known (N-1)/N) [3] and in this case is:

((N-1)/N =7/8 = 0.875);

d) taking into account the fact that the characteristics of TCS using Walsh codes. first of all, the level ($U(t)_{bmax}$) of the maximum (in terms of absolute value) side petals of the ACF is affected, it is proposed to consider in the future, first of all, only the petals with the level $|U(t)_{bmax}|$;

e) the above-mentioned Walsh-Harmuth code numbering system requires additional efforts when determining code numbers, therefore it is proposed (Fig. 6) to use code numbering according to the number of Hadamard matrix strips, which does not require additional time (as shown below, the proposed numbering even has a number of significant advantages over other numbering systems);

f) the results obtained by the software for ACF code 2 when the copy is shifted one bit to the right (Fig. 5) coincide with the results given in (Table 3), obtained graphically.

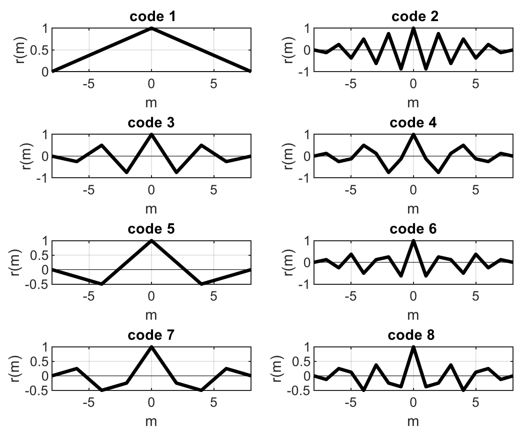

Fig. 6. ACF codes, as proposed numbering, (N=8)

Comparing the obtained ACFs (Fig. 5, Fig. 6) shows that their numbering is significantly different.

Using the proposed proposals for the primary analysis of the ACF of Walsh codes, the information about the ACF (Fig. 6) can be significantly simplified (Fig. 7), comparing it with its generally accepted presentation

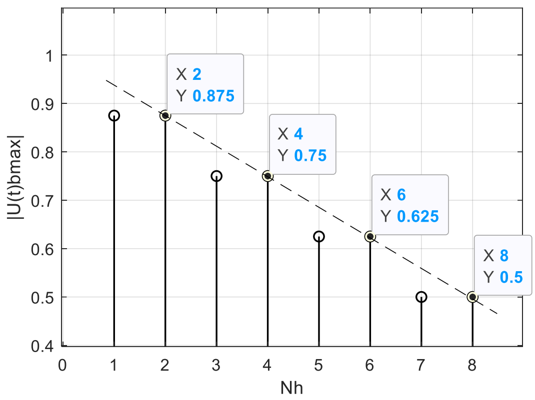

Fig. 7. The absolute value of the maximum values of the side lobes of the reinforced ACF of Walsh codes (N=8)

Based on the results (Fig. 7), it is possible to draw a number of important conclusions, which are problematic to obtain using a large array of data, such as (Fig. 5, Fig. 6):

a) the codes differ (or do not differ) in values $|U(t)_{bmax}|$, which can vary from (N-1)/N (for codes 1, 2) to (N-N/2)/N (for codes N -1, N) with a discrete value (discrete) Δ multiple of 1/N (in this case Δ=1/8=0.125);

b) an arbitrary code Nh with an odd number and the next code Nh +1 with an even number are characterized by the same value $|U(t)_{bmax}|$;

c) it is possible to optimize a complete array of codes (all codes are used) by turning it into an incomplete array (some codes are missing), thereby obtaining a decrease in $|U(t)_{bmax}|$ for an incomplete array.

For example, by converting the complete array of H=8 codes (Fig. 7) into an incomplete H12=8 (by removing codes 1 and 2), a decrease in |U(t)bmax| by 12.5% (87.5% - 75%). It is obvious that such optimization can be continued in the same way.

The obtained information (Fig. 7) can also be displayed in the Hadamard matrix (5) for this array of codes, supplementing it (Table 4) as follows.

Table 4

H_8								N_h	$\|U(t)_{bmax}\|$
1	1	1	1	1	1	1	1	1	
1	-1	1	-1	1	-1	1	-1	2	
1	1	-1	-1	1	1	-1	-1	3	
1	-1	-1	1	1	-1	-1	1	4	
1	1	1	1	-1	-1	-1	-1	5	
1	-1	1	-1	-1	1	-1	1	6	
1	1	-1	-1	-1	-1	1	1	7	
1	-1	-1	1	-1	1	1	-1	8	

In this way, an important result was obtained, which simplifies further primary analysis of the normalized ACF of Walsh codes:

a) generally accepted information contained in the Hadamard matrix (5) and generally accepted information (Fig. 5) can be displayed more compactly and visually;

b) for this, it is proposed to use the modified Hadamard matrix (Table 4).

It is of some interest whether the obtained conclusions regarding the absolute value of the maximum values of the side lobes of the normalized ACF of Walsh codes are preserved when their length increases (Fig. 8).

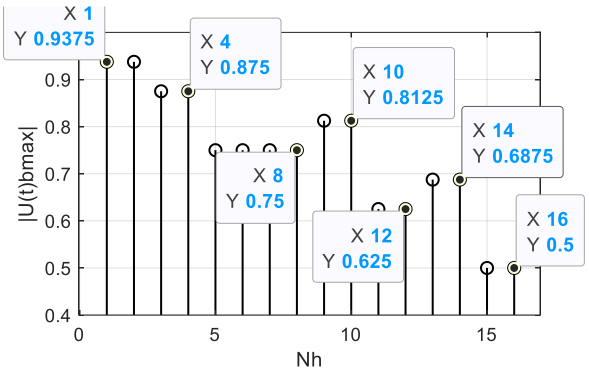

Fig. 8. The absolute value of the maximum values of the side lobes of the normalized ACF of Walsh codes (N=16)

It can be seen that the previously obtained conclusions are preserved regardless of the length of the Walsh codes. On the basis of the Hadamard matrix for this case, which was obtained similarly to matrix (5), and the data (Fig. 8), a modified Hadamard matrix was obtained for this case. A fragment of the specified matrix is shown in Table 5.

Table 5

H_{16}								N_h	$\|U(t)_{bmax}\|$
1	1	1	1	1	1	1	1..		
1	1	1	1	1	1	1	1		
1	1	1	1	1	1	1	1..		
-1	-1	-1	-1	-1	-1	-1	-1		
1	1	1	1	-1	-1	-1	-1		
-1	-1	-1	-1	1	1	1	1		
1	1	1	1	-1	-1	-1	-1		
1	1	1	1	-1	-1	-1	-1		
…									
							…		
1	-1	1	-1	-1	1	-1	1		
1	-1	1	-1	-1	1	-1	1		
1	-1	1	-1	-1	1	-1	1		
-1	1	-1	1	1	-1	1	-1		
1	-1	1	-1	1	-1	1	-1		
-1	1	-1	1	-1	1	-1	1		
1	-1	1	-1	1	-1	1	-1		
1	-1	1	-1	1	-1	1	-1		

But the data (analogous to Fig. 7, Fig. 8) for Walsh codes of length N=64 (Fig. 9) are the greatest interest, which are used in TCS of mobile communication with code division of channels (CDMA).

The data (Fig. 9) once again confirm the correctness of the previously formed conclusions obtained from the analysis of similar data (Fig. 7).

The analysis showed that the maximum level of the side lobes of normalized aperiodic ACF is far from optimal. It does not fall below the value $|U(t)_{bmax}|= 0.5$, and for most of the specified values the condition $|U(t)_{bmax}| \geq 0.75$. That is why the Walsh codes were not widely used in their "pure" form.

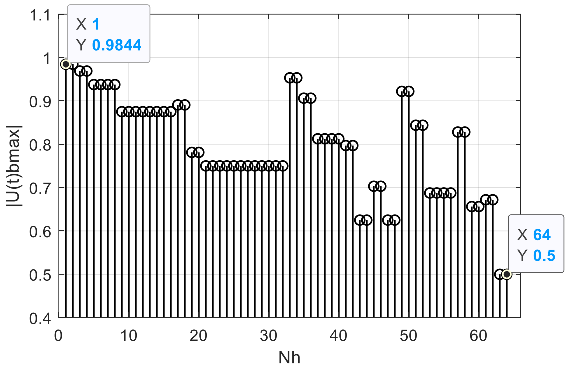

Fig. 9. The absolute value of the maximum values of the side lobes of the normalized ACF of Walsh codes (N=64)

Despite the indicated drawbacks of Walsh codes, the results obtained from the study of their ACF remain relevant. After all, regardless of the method of using Walsh codes (in a "pure" form or using scrambling), codes with better ACF indicators always have an advantage. The conducted studies indicated various methods of improving ACF indicators:

a) by using an incomplete array of codes of a given length;

b) increasing the length of the codes and forming an incomplete array from them.

IV. CCF WALSH CODES AND THEIR PROPERTIES

TCS indicators are affected by the properties of each of the correlation functions, both CCF and ACF. In fig. 9 shows the results of determining the normalized aperiodic CCF for some Walsh codes of length N=8.

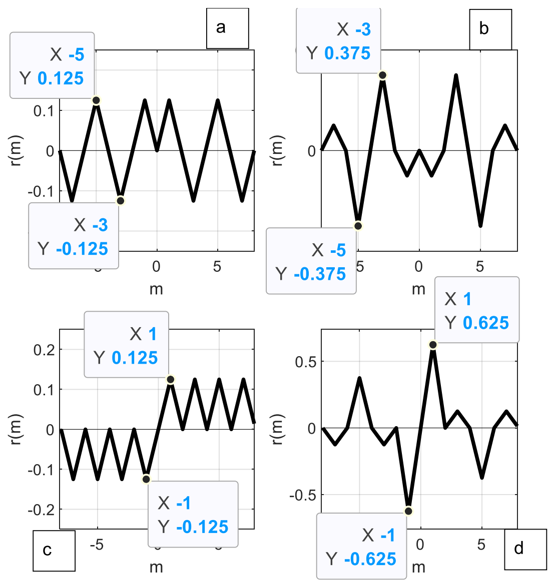

Fig. 10. Normalized aperiodic CCF of Walsh codes N=8: a) N_h = 3 and N_h = 2; b) N_h = 7 and N_h = 4; c) N_h = 1 and N_h = 2; d) N_h = 7 and N_h = 8.

Based on the data, similar (Fig. 10) but obtained for entire, a number of important conclusions can be drawn:

a) the main petal is zero, which indicates the orthogonality of Walsh codes;

b) CCF is not symmetrical relative to the main petal;

c) the range of changes in the level of the side petals is quite significant.

Similarly (Fig.10), the CCF was determined for all possible pairs of Walsh codes with N=8 (Table 6). At the same time, it is taken into account that Walsh codes, preserving their serial number N_h , can be obtained from the Hadamard matrix by reading them both "by rows" and "by columns" (see Table 4).

Table 6

N_h =1	N_h =2	N_h =3	N_h =4	N_h =5	N_h =6	N_h =7	N_h =8	N_h /CCF
1	1 1Δ	1 2Δ	1 1Δ	1 4Δ	1 1Δ	1 2Δ	1 1Δ	N_h =1 (CCF)
1 1Δ	-1	1 1Δ	-1 2Δ	1 1Δ	-1 3Δ	1 1Δ	-1 2Δ	N_h =2 (CCF)
1 2Δ	1 1Δ	-1	-1 **7Δ**	1 2Δ	1 1Δ	-1 4Δ	-1 3Δ	N_h =3 (CCF)
1 1Δ	-1 2Δ	-1 **7Δ**	1	1 1Δ	-1 2Δ	-1 3Δ	1 4Δ	N_h =4 (CCF)
1 4Δ	1 1Δ	1 2Δ	1 1Δ	-1	-1 3Δ	-1 6Δ	-1 3Δ	N_h =5 (CCF)
1 1Δ	-1 4Δ	1 1Δ	-1 2Δ	-1 3Δ	1	-1 3Δ	1 6Δ	N_h =6 (CCF)
1 2Δ	1 1Δ	-1 4Δ	-1 3Δ	-1 6Δ	-1 3Δ	1	1 5Δ	N_h =7 (CCF)
1 1Δ	-1 2Δ	-1 3Δ	1 4Δ	-1 3Δ	1 6Δ	1 5Δ	-1	N_h =8 (CCF)

Note. 1. The upper part of each row shows the value of a code with a specific number N_h , and the lower part of this row shows the value of the normalized aperiodic CCF of the selected code with another code (the number of which N_h corresponds to the column number in the selected cell of this row).

2. The CCF values differ by a multiple of $\Delta = 1/N = 1/8 = 0.125$.

3. If, for example, there is a value of 2Δ at the intersection of the row with the number N_h =5 and the column with the number N_h =3, this means that the value of CCF for the codes with the numbers 3 and 5 is $2\Delta = 2*\Delta = 2*0.125 = 0.25$.

CONCLUSION

Code sequences are widely used in TCS, such as broadband, and significantly affect the characteristics of systems as a whole. Walsh codes are often used as the indicated sequences. Along with their significant advantage - orthogonality of codes, they also have certain disadvantages - large levels of side lobes of normalized aperiodic correlation functions (both CCF and ACF).

When designing the TCS, the required codes are first of all subject to two mandatory requirements: the required length N of codes and the required number of them S. But when generating codes with a given value of N, their maximum number S is automatically generated, and N = S. Each of of the received codes may differ from others by the normalized value of the side lobes of the aperiodic correlation functions:

- from N/2 to (N-1)/N, for ACF;

- from 1/N to (N-1)/N, for CCF.

It is shown that when modifying the full array of codes S to a shortened array S1 < S, it is possible to significantly lower the level of side lobes of the correlation functions, which improves TCS indicators as a whole [5].

In this way, the possibility of optimizing the TCS is shown, which is proposed to be carried out using the data given in the modified Hadamard matrices (tables of the type table 4 and table 6 for codes N=8). As can be seen (Table 6), by optimally discarding only one code in the available array (N_h =4), it is possible to reduce the maximum level of the side petals of the CCF by 12.5% (from the level of 7Δ = 0.125*7=0.875 to the level of 6Δ = 0.125* 6=0.75). Similarly (Table 4), by optimally rejecting two codes (Nh =1 and Nh =2) in the existing array, it is possible to reduce the maximum level of the side petals of the VKF also by 12.5% (from the level of 0.875 to the level of 0.75).

The advantage of the proposed modified Hadamard matrices is the possibility of optimization based on compact matrices (such as Table 4 and Table 6 in the case of N=8) instead of using 64 graphic images (such as Fig. 6 and Fig. 10).

REFERENCES

[1] Tarek Kaddoura, Roger J. Zemp. Hadamard Aperiodic Interval Codes for Parallel-Transmission 2D and 3D Synthetic Aperture Ultrasound Imaging. Appl. Sci., Vol.12. 2022, https://doi.org/10.3390/app12104917.

[2] Z. Matousek, J. Perdoch, M. Pacek, S. Gazovova, J. Ochodnicky. Radar signal waveform based on Costas and Walsh-Hadamard codes as electronic counter-countermeasure. IET Radar, Sonar & Navigation. Volume 17, Issue 10, 2023.

[3] Agarwal P., Pratap A., Shukla M., “BER analysis of power rotational interleaver on OFDM-IDMA system over powerline”, In: Hemanth J., Bestak R., Chen J.IZ. (eds) Intelligent Data Communication Technologies and Internet of Things. Lecture Notes on Data Engineering and Communications Technologies, Vol 57, Springer, 2021.

[4] Maksymov V., Khrapovitsky I. Research of composite Barker codes // The scientific heritage, № 48 (2020), P.1, pp. 15-22.

[5] Klymash M.M, Pelishok V.O., Mykhailenych P.M. The method of signal formation the broadband wireless systems with code division of channels. Patent of Ukraine No. 77958, 2013.

Климаш М.М., Пелішок В.О., Михайленич П.М. Спосіб формування сигналу широкосмугових безпровідних систем з кодовим розділенням каналів. Патент України №77958, 2013.