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SOFTWARE ENGINEERING SECTION

inear Mathematical 3D Model of Determination mperature Field in Elements of Microelectronic Devices

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linear mathematical model for analysis of egimes in isotropic thermosensitive media with been developed. The temperature fields in such ue to a heat flux located at a boundary surfaces of the use of generalized function, Kirhoff's , and Fourier integral transformation, calculation determining the distribution of temperature in e structures with inclusions have been obtained. of heat conductivity of materials for such systems and according to cubic law depending on the

-3D medium; temperature regimes; heat conduction; n; thermosensitive system; heat

I. INTRODUCTION

ge of temperature fields in environment and in h time, the change of processes of operating in oelectronic devices depend on the effectiveness nass and exchange in heat. Therefore, during the the theory of heat exchange was intensively cause of requirements of heat power engineering, engineering space flights, etc. Nowadays, ways of against abnormal temperatures of supersonic esponsible zones of nuclear, power plants, in convert heat into electric power, in structures compressed gas. Heat exchange processes in law egimes, for example in plants with the use of tivity, in particular in magnetic substances which fields are being investigated. Works for creating tools for operations with quick freezing of imes of tissue are going on. The progress in this considerable extend relate to the due organizing of exchange of heat in a tool itself and in a ks for creation of plants for food drying with the mation high-quality development of which is the appropriate sublimation and desublimation, ducted.

The construction of solution (analytical and numerical) for problems of transfer of heat has practical, economic, and scientific significance. The temperature regimes for structure define to a great extent their qualitative and quantitative characteristics, and they are characterized by temperature field or by quantities which are determined from this fields, namely, the values of absolute temperatures, by temperature drops in space and in time, by behavior of temperature and of its gradients at boundary surfaces and at interfaces of different kinds of elements of structures with different given boundary conditions, by the time of occurring of the set distribution of temperature or of temperature drops, etc.

In some cases, mathematical modeling is the only means of information obtaining about fields of temperature in machine parts and assemblies of microelectronic devices, fields in elements of flying apparatus, elements of energy plants which are inaccessible to temperature sensors (temperature transducers) or to heat stream. Therefore, the construction of mathematical models of thermal state of different structures which are subjected to heating and cooling is of importance task because numerous quantitative, as well as qualitative indexes of estimation of behavior of structure which are more frequently determined considering regimes of temperature during their operation.

Composite materials, development of which is a liding problem of modern materials science, have become especially valuable in microelectronic devices. The creation of modern composite materials possessing improved physico-mechanical performance can promote the development of modern technologies in air-space, naval, power-generating, and electronic branches, in mechanical engineering. Structures containing foreign inclusions (they are widely used) occupied honorable place in technology, namely, in microelectronic devices, in integral sensors for monitoring of humidity and temperature, in elements which are light emitting, for dynamic LED-backlighting etc. Because the aforesaid structures operate in a wide range of temperatures, their high performances cause the necessity of considering and solving of the problems which are non-linear due to the dependence of thermophysical characteristics of the materials on the structure of the temperature, as well as on the heat exchange conditions, on their surface temperature; because finding the distribution of the temperature fields which are performed on the basis of the linear mathematical models of heat conduction processes do not always give us satisfactory results [1]. The main direction of development of modern sensor technology is the use of semiconductor materials, integrated technology, and on their basis the development of microelectronic converters. In particular, temperature transducers, which are the most important type of sensors, since most processes, including, those in everyday life, are regulated by temperature.

II. Analysis of literature sources and statement of the problem.

The determination of temperature regimes both in homogeneous and heterogeneous structures draws attention of many scientists [2].

In the paper [3], a mathematical model of calculation of a quasi-stationary temperature field in a continuous rotation cylinder of composite materials with non-linear boundary conditions in which the dependence of thermophysical parameters of materials is taken into account has been developed. The obtained analytical relationships for determination of temperature fields enable us to elaborate composite materials the composition for cylinder-shaped machines parts in order to increase their longevity.

1D (flat, cylindrically-symmetric and spherically-symmetric) non-linear problems of heat conduction for a given heat flux of the origin of coordinate system in the form of exponential function depending of time have been considered. Approximate solutions of the given problems have been obtained, their convergence has been analyzed [4].

An analytical-numerical solution of the non-linear problem of heat conduction with the use of the integral method of heat balance has been found [5]. In order to improve the accuracy of the solution, the temperature function is approximated by high-degree polynomials. To determine the coefficients of the polynomials, additional boundary conditions have been introduced. It is shown that such an approach leads to considerable improvement in accuracy of the solution of the problem as early as in the second stage of approximation.

In the paper [6], an analytical numerical solution of the non-stationary (non-steady-state) problem of heat conduction for a hollow ball, the thermophysical parameters of the material of which are dependent on temperature is obtained. In a particular case, the solution for a continuous ball has been obtained.

Variational approach for development of a non-linear mathematical model of the heat conduction process for a 2D medium with thin inclusion has been applied. For linearization of the formulated problem, the Newton-Ruffson's method has been applied. The discretization with respect to time variable has been carry out according to the scheme of intermediate point [7].

In the paper [8], the non-steady-state problem of heat conduction for functional spheres has been solved. Thermophysical and thermoelastic parameters of the [illegible] excepted Poisson ratio, are arbitrary function [illegible] coordinate.

The axial-symmetric about the heat source [illegible] problem of heat conduction and thermo elasticity [illegible] functional-gradient spheres has been considered. [illegible] the form of functions of spatial coordinates for [illegible] components of the displacements vector and of the [illegible] have been obtained with the use of boundary [illegible] respect to the radial and angular coordinates [9].

The review of main literature sources in the [illegible] development and investigation of mathematical [illegible] conduction process has indicated that models which [illegible] into account piece-wise homogeneity of structures [illegible] thermosensitivity (dependence of thermophysical [illegible] on temperature) still remain to be but poorly [illegible] non-developed. Since such structures are [illegible] temperature impacts, in certain ranges of [illegible] influence of thermosensitivity on results of [illegible] temperature fields becomes to be considerable.

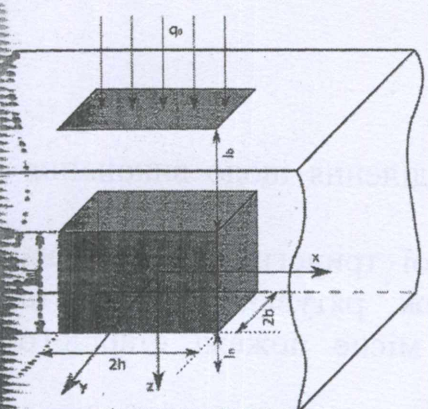
The investigation of the operation of any [illegible] is accompanied by physical or mathematical [illegible] physical modeling is impossible because of the [illegible] complexity of the device, for example, the [illegible] large and very large integrated circuits. In these [illegible] resorts to mathematical modeling using the means and [illegible] of computer technology.

Calculations of temperature regimes in [illegible] microelectronic device are to be further used for [illegible] complicated systems in order to ensure their [illegible] the precision of such calculations will [illegible] effectiveness of designing methods [10].

The boundary value non-linear problem of heat [illegible] is determined below, the technique of its [illegible] calculation formulae for determination of the [illegible] in a heat-sensitive plate with a through inclusion [illegible] heated by a heat stream concentrated at a surface [illegible] inclusion is given. Numerical analysis under the [illegible] linear dependence of the coefficient heat [illegible] temperature is performed.

III. Object of investigation and mathematical [illegible]

The 3D medium represented by isotropic layer [illegible] contains parallelepiped shaped inclusion whose [illegible] $V_0 = 8hbd = 2dS_0$ is considered. At one of the layer's [illegible] $K_b = \{(x, y, -d - l_b) : |x| < \infty, |y| < \infty\}$, there is a [illegible] heat flow whose surface density is equal to $q_0 = const$, [illegible] other surface $K_n = \{(x, y, d + l_n) : |x| < \infty, |y| < \infty\}$, there [illegible] the conditions of convective heat exchange with the [illegible] medium at the constant temperature of $t_c = const$ (fig. [illegible] the surfaces of the inclusion, there exist conditions of [illegible] contact. The presented structure is referenced to the [illegible] coordinate system (x, y, z) whose origin O is located [illegible] center of the inclusion (fig. 1).



...e 1. Isotropic layer with inclusion

...e stationary temperature field $t(x,y,z)$ in ...homogeneous medium, the equation of ...ned

$$...[\lambda(x,y,z,t) grad\ t(x,y,z)]=0, \quad (1)$$

$$...(\lambda_0(t)-\lambda_1(t))N(x,h)N(y,b)N(z,d); \quad (2)$$
$$Q(x,y)=q_0 N(x,h)N(y,b);$$

...coefficient of thermal conductivity of the ...temperature-sensitive layer; $\lambda_1(t),\lambda_0(t)$ are ...mal conductivity for the materials of the ... the inclusion, respectively; ...$-\eta)$

$\zeta>0$
$\zeta=0$ is the symmetric unit function.
$\zeta<0$

...ditions are of the form:

$$\lambda(t)\frac{\partial t}{\partial z}\Big|_{z=-d-l_b}=-Q(x,y),$$

$$...\Big|_{|x|\to\infty}=t\Big|_{|y|\to\infty}=0. \quad (3)$$

...med that the size of the inclusion are small in ... with the distances l_b, l_n from its boundary ...$=\{(x,y,\pm d):|x|\le h,|y|\le b\}$ to the boundary ...K_n of the layers. Let us introduce the reduced ...tivity $\Lambda_0(t)=\lambda_0(t)V_0$ of the inclusion, reduced ...sity $Q_0=q_0S_0$ of the flow; and then let us take limit ...ions (2) for $h\to 0, b\to 0, d\to 0$; Λ_0 and Q_0 ... and the known limit $\lim\limits_{\eta\to 0}\frac{N(\zeta,\eta)}{2\eta}=\delta(\zeta)$ being ...lt, we obtain relationships

$$...(x,y,z,t)=\lambda_1(t)+\Lambda_0(t)\delta(x,y,z),\ Q(x,y)=Q_0\delta(x,y)$$

with allowance for which, after certain transformations, we rewrite equation (1) in the form

$$div\left[\lambda_1(t) grad(x,y,z)\right]+$$
$$+\Lambda_0\frac{\partial t(0,0,z)}{\partial z}\Big|^{*}_{z=0}\delta(x,y)\delta'(z)=0. \quad (4)$$

Here $\delta(x,y,z)$ is the Dirac delta function.

Consider the Kirchhoff variable

$$\vartheta(x,y,z)=\frac{1}{\lambda_1^0}\int_0^{t(x,y,z)}\lambda_1(\zeta)d\zeta, \quad (5)$$

which differentiates between variables x, y and z, we obtain

$$\lambda_1^0\frac{\partial\vartheta}{\partial\zeta}=\lambda_1(t)\frac{\partial t}{\partial\zeta} \quad (\zeta=x,y,z), \quad (6)$$

where λ_1^0 is the reference coefficient of thermal conductivity of the layer.

Using expression (6), the equation (4) is transformed and written in the form

$$\Delta\vartheta=-\frac{1}{\lambda_1^0}\left[\Lambda_0\frac{\partial t(0,0,z)}{\partial z}\Big|^{*}_{z=0}\delta'(z)\right]\delta(x,y). \quad (7)$$

The relationship (5) enables us the boundary conditions (3) to be written in the form

$$\frac{\partial\vartheta}{\partial z}\Big|_{z=d+l_n}=0,\ \vartheta\Big|_{|x|\to\infty}=\vartheta\Big|_{|y|\to\infty}=0,\ \frac{\partial\vartheta}{\partial z}\Big|_{z=-d-l_b}=-Q(x;y). \quad (8)$$

By means of the application of the integral Fourier transformation with respect to the coordinates x and y to the problem (7), (8), we obtain an ordinary differential equation with constant coefficients

$$\frac{d^2\bar\vartheta}{dz^2}-\gamma^2\bar\vartheta=-\frac{1}{2\pi\lambda_1^0}\left[\Lambda_0\frac{\partial t(0,0,z)}{\partial z}\Big|^{*}_{z=0}\delta'(z)\right] \quad (9)$$

and such boundary conditions:

$$\frac{d\bar\vartheta}{dz}\Big|_{z=d+l_n}=0,\quad \frac{d\bar\vartheta}{dz}\Big|_{z=-d-l_b}=-\frac{Q_0}{2\pi\lambda_1^0}, \quad (10)$$

where

$\bar\vartheta(\alpha,\beta,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\alpha x}dx\int_{-\infty}^{\infty}\vartheta(x,y,z)e^{i\beta y}dy$ is a transformant of the function $\vartheta(x,y,z)$.

The general solution of equation (9) is obtained by the method of variation of constants in the form

$$\bar\vartheta(\alpha,\beta,z)=c_1e^{\gamma z}+c_2e^{-\gamma z}-\frac{1}{2\pi\lambda_1^0}\left[\Lambda_0\frac{\partial t(0,0,z)}{\partial z}\Big|^{*}_{z=0}\right]\mathrm{ch}\,\gamma z\ S(z).$$

Using the boundary conditions (10), we define the integration constant. As a result, we obtain a partial solution of the problem (9), (10):

$$\bar\vartheta(\alpha,\beta,z)=\frac{1}{\pi\lambda_1^0}\left\{0,5\Lambda_0\frac{\partial t(0,0,z)}{\partial z}\Big|^{*}_{z=0}\left[\frac{ch\gamma(z+d+l_b)}{sh\gamma(2d+l_n+l_b)}sh\gamma(d+l_n)-\right.\right.$$
$$\left.-ch\gamma z S(z)+\frac{Q_0}{\gamma}\left[\frac{ch\gamma(z-d-l_n)}{sh\gamma(2d+l_n+l_b)}\right]. \quad (11)$$

Applying the inverse Fourier transform to the relationship (21), we obtain an expression for the function ϑ

$$\vartheta(x,y,z)=\frac{2}{\pi}\int_0^\infty\int_0^\infty \cos\alpha x\cos\beta y\,\bar{\vartheta}(\alpha,\beta,z)\,d\alpha\,d\beta. \qquad (12)$$

Using relations (5), (12) for the existing dependences of the coefficient of thermal conductivity of the layer material on temperature, we obtain a nonlinear equation for determining the value

$$\left.\frac{\partial t(0,0,z)}{\partial z}\right|_{z=0}^{*}.$$

The desired temperature field in the reduced inhomogeneous thermosensitive spatial system is determined from the nonlinear equation obtained using relations (5), (12) and the existing dependences of the coefficient of thermal conductivity of the material of the layer on the temperature.

IV. PARTIAL EXAMPLES AND ANALYSIS OF THE RESULTS

For low temperatures, the dependence of the thermal conductivity of nonmetallic crystals on temperature is expressed in the form [11]

$$\lambda(t)=\kappa t^3 \qquad (\kappa - const). \qquad (13)$$

Then using the expressions (5) and (12) we obtain a formula for determining the temperature *t (x, y, z)*

$$t(x,y,z)=\sqrt[4]{4/\kappa\lambda_1^0\,\vartheta(x,y,z)}. \qquad (14)$$

The value of value

$$\left.\frac{\partial t(0,0,z)}{\partial z}\right|_{z=0}^{*}$$

we define from such a nonlinear equation:

$$\left[4\vartheta(0,0,0)\right]^{\frac{3}{4}}\left.\frac{\partial t(0,0,z)}{\partial z}\right|_{z=0}^{*}-\left(\frac{\lambda_1^0}{\kappa}\right)^{\frac{1}{4}}\left.\frac{\partial\vartheta(0,0,z)}{\partial z}\right|_{z=0}=0.$$

In many practical problems there is such a dependence of the coefficient of thermal conductivity on temperature [11]:

$$\lambda(t)=\lambda^0(1-kt), \qquad (15)$$

where λ^0 and k are the reference and temperature coefficients of thermal conductivity.

Using the relationships (5), (12), we obtain the following formula for determining the temperature *t (x, y, z)*:

$$t(x,y,z)=\frac{1-\sqrt{1-2k_1\vartheta(x,y,z)}}{k_1}, \qquad (16)$$

but the quantity

$$\left.\frac{\partial t(0,0,z)}{\partial z}\right|_{z=0}^{*}$$

is determined from the nonlinear equation

$$\left[\left.\frac{\partial\vartheta(0,0,z)}{\partial z}\right|_{z=0}\right]^2-\left[1-2k_1\vartheta(0,0,0)\right]\left[\left.\frac{\partial t(0,0,z)}{\partial z}\right|_{z=0}^{*}\right]^2=0,$$

where

$$\left.\frac{\partial\vartheta(0,0,z)}{\partial z}\right|_{z=0}=\frac{1}{\pi^2\lambda_1^0}\left\{Q_0\int_0^\infty\int_0^\infty\left[\frac{sh\gamma(d+l_n)}{sh\gamma(2d+l_n+l_b)}ch\gamma(d+l_n)-\frac{1}{2}\right]d\alpha\,d\beta-\right.$$
$$\left.-\Lambda_0\left.\frac{\partial t(0,0,z)}{\partial z}\right|_{z=0}^{*}\int_0^\infty\int_0^\infty\gamma\frac{sh\gamma(d+l_n)}{sh\gamma(2d+l_n+l_b)}sh\gamma(d+l_b)\,d\alpha\,d\beta\right\};$$

$$\vartheta(0,0,0)=$$
$$=\frac{1}{\pi^2\lambda_1^0}\left\{\Lambda_0\left.\frac{\partial t(0,0,z)}{\partial z}\right|_{z=0}^{*}\int_0^\infty\int_0^\infty\left[\frac{1}{2}-\frac{ch\gamma(d+l_n)}{sh\gamma(2d+l_n+l_b)}sh\gamma(d+l_b)\right]d\alpha\,d\beta\right.$$
$$\left.+Q_0\int_0^\infty\int_0^\infty\gamma\frac{ch\gamma(d+l_n)}{\gamma\,sh\gamma(2d+l_n+l_b)}ch\gamma(d+l_b)\,d\alpha\,d\beta\right\}.$$

Thus, we obtain convenient formulas (14), (16) determining the temperature field caused by the heat which makes it possible to analyze the temperature regim a nonhomogeneous thermosensitive spatial structure.

V. CONCLUSIONS

A non-linear mathematical model for determinatio caused by heat flow temperature field for structures which been geometrically described by a thermosensitive iso layer with an inclusion has been development. The enable us to analyze temperature regimes in thermosen media with inclusions. Results of the analysis can be used prognosing the regimes of operation of systems of geometrical structure, as well as to identify the unkn parameters and to improve thermoresistance, that incre their longevity.

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