**Computational structure of DST-II using convolvers**

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*Abstract***The computational structure of discrete sine transform (DST) using convolvers is considered. Hashing arrays on the basis of simplified arguments of basis DST of type II for size N=2n, 3n and computational structure are analyzed,** **what eliminate the analysis of the identity the cyclic submatrices in the basis matrix of DST-II transforms.**

*Кеу words*  **discrete sine transform, hashing array, computational structure, convolver, transform size.**

I. Introduction

Discrete transforms and convolutions are main operations and key tools in digital signal processing. The discrete sine transform (DST) [1] with a real basis reproduces functionality in space or time dependence similar to the DFT. DST and DFT interrelate strict mathematical forms that allow finding an effective way to compute one transform through another. The basic DST functions are well approximated to a big number of stationary stochastic processes, which allows describing the signal with a given accuracy of minimal number of components. DST is used in many applications, especially in the processing of digital audio and video signals [2].

There are 8 types of DST I-VIII discrete sine transform discussed in [1]. Information technologies widely use type DST-II, which can be represented respectively by the following formula

 (1)

where α(n)=1/√2 , if *n*=0; otherwise α(n)=1, k=0,1,2,…,N-1.

Further intensive development of information technologies puts higher demands on performance, functionality and specific opportunities for algorithmic, software and hardware of the real discrete transforms. The successful use of DST relies on the existence of the so-called fast transforms. A significant event associated with the trend of development the efficient algorithms it is possible the computation of DST through cyclic convolution among others approaches. The technique, first used by Rader for obtaining a prime length of DFT [3], identifies cyclic structures within the transform matrix. A lot of the papers [4-6] appeared about efficient hardware architectures connected with the computation through cyclic convolution. Using low complexity of convolution algorithms [7] in block-cyclic structures of basis matrix leads us to efficient computation of transforms of Fourier class.

II. The Computational Method of DST-II on Basis of Cyclic Convolutions

The approach of the synthesis the algorithms for DST-II requires analyzing the features of basis transform [8]. The determination of the block-cyclic structure the matrix of arguments the basis of transform define the efficient computation of DST-II the real sequences of arbitrary size. The matrix of integer arguments the basis of the DST-II transforms of simple or composite size N effectively represents the set of Hankel circulant submatrices. The structure of the matrix of arguments the basis of DST-II is compact described by the corresponding cyclic decomposition of substitution (hashing array P(n)). A difference of values Eq.(1) in rows (k+1) and columns (2n+1) requires transition from hashing array P(n) to the appropriate hashing arrays indices of the rows Pr(n) and columns Pc(n). Basis matrix of arguments DST-II with integer elements contains the values (2n+1) of elements in the first row, which cover the entire period equal to 4N, and k of the elements in the first column, which cover the period equal to 4N. The decomposition of the structure the basis of DST-II on Hankel submatrices requires rearrangement of rows and columns in accordance with the hashing array Pr(n), Pc(n).

As a result, based on this approach [9], the basic structure of the matrix is specifed a hashing array represented in the form

Pr(n)=Pr(n1) Pr(n2) …Pr(nk), (2)

Pc(n)=Pc(n1) Pc(n2) …Pc(nr), (3)

where n – size of hashing array, ni – size of hashing subarray P(ni), k,r – number of hashing subarrays.

The hashing array Pr(n), Pc(n) for the property of the symmetry the basic functions can be set using the simplified hashing arrays Pr’(n), P’c(n) supplemented by a respective arrays of signs Sr(n), Sc(n):

Pr’(n)=Pr’(n1)Pr’(n2) …Pr’(nk), (4)

 Sr(n)=Sr(n1)Sr(n2) …Sr(nk), (5)

Pc’(n)=Pc’(n1)Pc’(n2) …Pc’(nr), (6)

 Sc(n)=Sc(n1)Sc(n2) …Sc(nr), (7)

where the Sr(n), Sc(n) contain elements that can be equal to + 1, −1, 0 (indicate + , − ,0 for short).

For a given size N of DST-II form the hashing arrays Pr(n), Pr’(n), Sr(n) and Pc(n), Pc’(n), Sc(n) and determine the features. These specific parameters and features include:

- N values of transform size N = p (simple or odd ); N = pn; N = 2n; N = 2npn; n=0,1,2,…

- k,r - number of subarrays in hashing array;

- Li - number of elements in hashing subarray;

- number of submatrices in the base matrix structure;

- coordinates (i,j) with the corresponding sign sij and simplified value cij of first elements of the submatrices in matrix structure;



Fig. 1.The computational structure of DST using the convolvers.

- number the retrying of identical submatrices for horizontally/vertically r<m in a matrix structure;

- retrying of elements in a row for the submatrices corresponding of hashing subarray;

- property the cyclical left of the submatrices (Hankel), formed by the corresponding hashing subarray.

Analysis the identity of cyclic submatrices in the structure of the base the discrete sine transform makes more efficient computing algorithm. The definition the identity of cyclic submatrices understood the search of the identity and quasi identity submatrices (have the same simplified elements, but the opposite meaning signs). Indeed, the horizontal and vertical analysis of a identity and the selection the cyclic submatrices of the arguments and signs reduces the number of execution cyclic convolutions. In result the horizontal and vertical analysis of cyclic submatrices in basis matrix determines the number of cyclic convolutions, which are executed in computational structure using convolvers.

As a result, the advantages of the algorithm of computation of DST-II based on cyclic convolutions:

- a general method of using hashing array Pr(n), Pc(n) to arrive at an efficient conversion on the basis of an arbitrary length DST into parallel circular structures;

- the hashing array Pr(n) corresponds to the cyclic decomposition of substitution of columns from basis matrix of arguments, that empower the usage of Pr(n) and Pc(n), as part of Pc(n), for the synthesis of efficient algorithms of DST-II on the basis of cyclic convolutions;

- analysis of the level of simplified hashing array Pr(n) with supplement of respective subarray of Sr(n) signs reduces the number of computation of cyclic convolutions, because the set of argument values of the basis matrix is reduced;

- the synthesis of algorithms including determination of Pr(n), Pr′(n), Sr(n) and analysis of the structure of basis matrix uses integer arithmetic.

III. The Computational Structure of DST-II Using the Convolvers

There are different approaches to implementation of the DST. Traditional implementations use the butterfly structures [10], which are not suitable for efficient hardware implementation because they require global communications. That imposes limitation on the computational rate when the number of processors becomes large and their common area increase. The systolic array structures in the convolvers [5,6] are suitable for parallel computation of digital signal processing algorithms. A solution to allow local data exchanges between processing elements, that consists of locally connected processors. A simultaneous data flow via local connections does not require any control overhead.

The approach of the implementation of DST on basis convolvers can significant saves the average value of the delay-area product (gate-count, time-unit) using on a modern integrated technology. The resulting computational structure of DST using the convolvers consists such components (Fig.1): input buffer memory, data vector adder - VA, convolver, multiplex switch, output buffer, union of vector input data - X[Pi(ni)], vector data of DST coefficients -W[Pi(ni)], ni - the size of cyclic convolution.

 The computational algorithm of DST using cyclic convolutions consists such sequential stages:

- unions of input data vectors x(k) according to identical horizontal submatrices;

- computation of cyclic convolutions using efficient algorithms of fast cyclic convolutions;

- unions of the intermediate results of cyclic convolutions to output X(k) data.

The first and the last computational stages are simple data vector additions in the VA. A number of the data vector additions

are determined of identity submatrices in the analysis of the structure of the base matrix.

Consider these stages using generalized scheme for synthesis of efficient DST-IІ algorithms [9] for the examples sizes of N=2n and N=3n. The block-cyclic structure of basis matrix of DST-IІ can be set as a hashing array Pr(n), Pc(n) for size N=8 in the versions a), b).

a) Pr(16)= (**1**,**3**,9,**27**,17,19,**25**,11) (**2**,**6,**18,22)(**4**,12)(**8**);

P’r(16)= (1,3,7,5,1,3,7,5)(2,6,2,6)(4,4)(8),

Sr(n)= (+,+,+,-,-,-,-,+)(+,+,-,-)(+,+)(+);

Pc(8)= (1,3,9,27,17,19,25,11); P’c(8)= (1,3,7,5,1,3,7,5), Sc(n)= (+,+,+,-,-,-,-,+).

b) Pr(16)= (**1**,**7**,17,23) (**3**,21,19,**5**) (**2**,14) (**4**,28)(**6,**10)(**8**),

Pr’(n)= (1,7,1,7) (3,5,3,5) (2,2) (4,4)(6,6)(8),

Sr(n)= (+,+,-,-)(+,+)(+,-,-,+)(+,-)(+,+)(+);

Pc(8)=(1,7,17,23)(3,21,19,5);

 P’c(8)=(1,7,1,7)(3,5,3,5), Sc(n)= (+,+,-,-)(+,-,-,+).

The time delay ti of computation the output data X(i) i=1,2,…,8) of DST-II accordance to the structure (Fig. 1) are determined:

a) t1,3,5,7 =ta + t4p; b) t1,7 = t3,5= ta + t2p + ta;

 t2,6 =2ta + t2p ; t2,6 =2ta + t1p + ta;

 t4 =3ta + t1p ; t4 =3ta + t1p ;

 t8 =4ta ; t8 =4ta ,

where ta - time delay of the data vector adder; tkp- time delay of the k- point cyclic convolution of the convolvers. The unions of the intermediate results of cyclic convolutions not perform, in case of the version a) the hashing array Pc(n) consists one subarray.

The resulting P(2N-1) hashing array of DST-II for sizes N=2n (2) is represented in the Table 1.

TABLE 1

RESULTING HASHING ARRAYS OF DST-II FOR N=2n

|  |  |
| --- | --- |
| Type | Resulting hashing arrays |
| ДCП-II | a) Pr(2N-1)= **P(2n)** P(2n-1)P(2n-2)... P(21)P(20);b) Pr(2N-1)=**P(2n-1)P(2n-1)**P(2n-2)P(2n-2)... P(20); c) …  |

The hashing subarrays with the odd elements (bold) of the hashing array of the rows Pr(n) correspond the hashing array of the columns Pc(n). The sizes of cyclic convolutions are equal ni=2n (n=1,2,3,…,n) for of DST-II the value of sizes integer power of two (Fig. 1).

The block-cyclic structure of basis matrix of DST-II defines the hashing arrays Pr(n), Pc(n) for size N=32,33 in form of the versions: for DST-II, N=32=9,

a) Pr(18)= (**1**,**5**,**25**,17,13,29) (**3**,15) (**9**) (**2**,10,14,34,**26,22**) (**6**,30)(18),

P’r(18)= **(1,5,7,1,5,7) (3,3) (9)** (2,8,4,2,8,4)(6,6)(0),

Sr(18)= (+,+,-,+,+,-)(+,+)(+) (+,+,+,-,-,-) (+,-)(0);

P’c(9)= (1,5,7,1,5,7)(3,3)(9), Sc(9)= (+,+,-,+,+,-)(+,+)(+);

for DST-II, N=33 =27,

a) Pr(n)=(**1**,**5**,**25**,**17**,85,101,73,41,97,53,49,29,37,77,61,89,**13**, 65) (**3**,**15**,75,51,39,87)(**9**,45)(27)

(**2**,**10**,50,34,62,94,38,82,86,106,98,58,74,46,**14**,70,**26**,**22**) (**6**,30,42,102,78,66)(**18**,90)(0);

P’r(n)=(1,5,25,17,23,7,19,13,11,1,5,25,17,23,7,19,13,11) (3,15, 21, 3,15,21)(9,9)(27)

(2,10,4,20,8,14,16,26,22,2,10,4,20,8,14,16,26,22)(6,24,12,6, 24,12)(18,18)(0),

Sr(n)=(+,+,+,+,-,-,-,+,-,+,+,+,+,-,-,-,+,-)(+,+,-,+,+,-)(+,+) (+1)

(+,+,+,+,-,-,+,-,-, -,-,-,-,+,+,-,+,+) (+,+,+,-,-,-)(+,-)(0);

b) Pr(n)=(1,11,13,35,47,23,37,25,49)(5,53,43,41,19,7,31,17, 29)(3,33,39)(15,51,21)(9)(27)(45) (2,22,26,38,14,46,34,50,10)(4,44,52,32,28,16,40,8,20) (6,42,30) (12,24,48) (18) (36) (0);

P’r(n)=(1,11,13,19,7,23,17,25,5)(5,1,11,13,19,7,23,17,25) (3,21,15)(15,3,21)(9)(9)(27)

(2,22,26,16,14,8,20,4,10)(4,10,2,22,26,16,14,8,20)(6,12,24) (12,24,6) (18)(18)(0);

Sr(n)=(+,+,+,-,-,+,-,+,-)(+,-,-,-,+,+,-,+,-)(+,-,-)(+,-,+)(+)(+)(+)

(+,+,+,-,+,-,-,-,+)(+,-,-,-,-,+,-,+,+)(+,-,-)(+,+,-)(+)(-)(0).

The resulting P(2N-1) hashing array of DST-II for sizes N=3n with the number of elements in the round brackets in versions a) of the hashing subarrays represented in the form:

a) P(2∙3n-1)=**P1(2\*3n-1)P2(2\*3n-2)P3(2\*3n-3)... Pn(2\*30)Pn+1(1)** Pn+2(2\*3n-1) Pn+3(2\*3n-2) Pn+4(2\*3n-3)...P2n-1(2\*30) P2n(1); (8)

The convolvers (Fig.1) could perform such dimensions of the cyclic convolutions: 2\*3n-1, 2\*3n-2, …,2\*32, 2\*31, 2, 1 (n=1,2, 3,…) for computation of DST-II of the sizes N=3n integer power of three.

IV. Conclusion

The algorithms of DST using cyclic convolutions and the possibility of the performance of this method for computational structure using the convolvers arepresented. The paper involves defining parameters of hashing array of DST-II for N=2n, 3n, what eliminate the analysis of the identity the cyclic submatrices in the basis matrix of transform. Separate computations of cyclic convolutions, which are structured according to this approach to the basis matrix, and the combinations of results make the proposed technique vital for concurrent programming and for implementation in parallel systems. The cyclic convolutions can be realized through very simple hardware structure - convolvers, which is very suitable for VLSI implementation [5,6].

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