MATHEMATICAL MODELING OF THE EARTH HEAT PROCESSES FOR THE PURPOSES OF ECO-TECHNOLOGY AND CIVIL SAFETY

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*T*he article suggests a new model of the Earth's thermal processes, based on a new understanding and formalization of cumulative processes of heating and cooling of parts of the Earth (inner core, outer core and mantle, crust, atmosphere). The geothermal model of the Earth with heat fluxes constructed according to the laws of thermodynamics, in which from a depth near to the center of the Earth's core there is a constant power coming from a depth to the surface of the Earth. One of the consequences of the new model of the Earth's thermal processes is the justification of the existence of an internal source of heat in the center of the Earth's core, from which an energy is absorbed in a small part by its substance and, in the larger part, is emitted into the open physical space. Mathematical modeling of the Earth heat processes formulate a new understanding of the thermal processes in our planet in general and is the basis for creating tools for predicting dangerous natural processes of endogenous origin, making possible the stability analyzing of natural geosystems, developing and new eco-geotechnologies.

Keywords — *geothermal model of the Earth; forecasting emergencies; natural ecosystems; civil protection*

I. INTRODUCTION

Predicting emergencies of natural endogenous origin at the current stage of science is unreliable. One of the reasons for the low reliability of such forecasts is the unresolved fundamental issues of the Earth's thermal processes. The analysis of the developed geothermal model of the Earth with heat fluxes indicates the increase of the planet's temperature with constant total gravitational energy with increasing depth. Alongside in the model have place the existence of the physical sequential mechanisms that are triggered by the thermal energy when approaching the Earth's center with the increase of the thermal energy density in the center of the Earth's core.

The present state of the Earth is filled with various threats of emergencies of natural and man-made origin. If man-made threats of emergencies are widely investigated, alongside natural threats are much less studied.

Annually, there are approximately about 1200 earthquakes in the world with magnitude over 6 points. Over the past 30 years, there has been a tendency for an increase of ruining earthquakes with magnitudes greater than 6 points (Fig. 1).
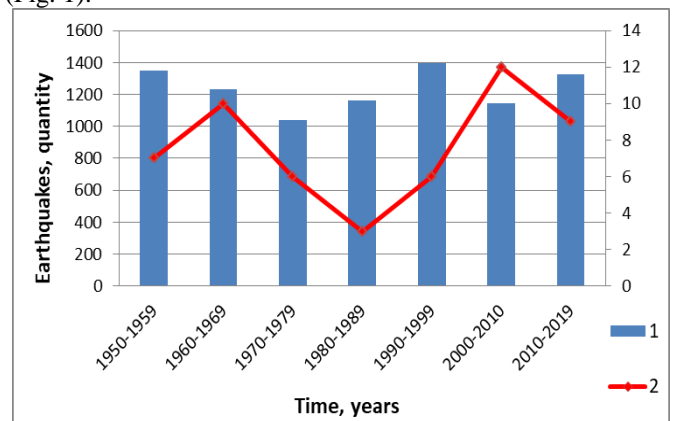

Fig. 1. Dynamics of earthquakes in the world: 1 - the number of all earthquakes, 2 - the number of earthquakes with a magnitude greater than 6 points [1]

Accordingly, there are increasing material losses and human casualties as a result of earthquakes. The record year was the 2010 earthquake that killed 226,000 people [1]. One of the reasons for such great human casualties from the impact of earthquakes is the poor accuracy and reliability of their prediction. The reliability of predicting endogenous activity is so low that the United State Geological Survey use information from Twitter popular service information site.

The main problem of predicting emergencies of natural endogenous origin is white spots in understanding of the

Earth's thermal processes. It is impossible to predict the occurrence of an earthquake without a reliable model of thermal processes in insides of the Earth, those processes determine the motion of lithospheric plates, and smaller elements of the Earth's crust. Also, a poor understanding of thermal processes leads to emergencies in mining [2] and delay the development of eco-technologies.

II. STATE OF THE STUDY OF THE EARTH'S HEAT PROCESSES

A number of scientists investigated a problem of the geothermal model of the Earth. In particular, Sorohtin O., Ushakov S. [3] have proposed geothermal models of "cold" Earth. The authors of this model consider the process of cooling the Earth with radioactive sources of internal heat as a conductive heat flux, the temperature in the center of the Earth is constant throughout its evolution, about - 6370 ° K.

Kuznetsov V.V. [4] have proposed a model of "hot" Earth. According to this model, the conductive heat flux of the Earth occurs as a result of phase transformations of matter on the surface of the nucleus. He go out from the initial temperature in the center of the Earth at the level of 30000°K.

Despite the significant contributions of these and other researchers, some fundamental issues remain unresolved. In particular, why is the power density (398.2 W/m^2) of radiate heat flux from the Earth's surface 4577 times greater than the power density (0.087W/m^2) of its conductive heating-cooling, and at a depth of 50 m (364.46W/m^2) – 4189 times? What heat fluxes are the cause, and what are the consequences in the geothermal Earth's model, and from which physical processes or from one common physical process? This contradiction shapes the purpose and objectives of the presented study.

III. AIM OF THE RESEARCH

Develop and investigate a dynamic geothermal model of the Earth with exogenous and endogenous heat fluxes observed on the Earth's surface at given values of their thermal energy power, elastic parameters distribution in the nucleus, mantle, crust, and temperature of the atmosphere. The model will create a new fundamental basis for a variety of practical applications, including an improvement of the reliability predicting dangerous natural processes of endogenous origin.

IV. RESULTS OF THE RESEARH AND DISCUSSION

In general, the intensity of the Earth's heat fluxes depends on the energy of the Sun, the parameters of the atmosphere, the lithosphere, the mantle, and the core of the Earth. The geothermal model of the Earth (GTME) with heat fluxes is constructed according to the laws of thermodynamics, in which the Earth's internal heat source (EIHS) has a constant power of $1,859*10^{17}$W ($364,46$ W/m^2), which extends from a depth of 50 m to the Earth's surface. The model works as follows: EIHS heat the inner core, which cool, heating the outer core and mantle. The cooling of the mantle is accompanied by the heating of the lithosphere. And the cooling of the lithosphere is due to the heating of the atmosphere, which cool with a radiation flux density of 239.9 W/m^2 of average infrared (IR) waves [5]. Analysis of the Earth's heat balance shows that the atmosphere transmits 66% of the average IR waves of the EIHS. Thermal flux of short absorbed IR waves penetrates the Earth's surface at $0.41*10^{17}$ W (161W/m^2) [6], (163.3 W/m^2) [5]. The heat flux density of 398.2 W/m^2 [5] from medium and long IR waves corresponds to an average temperature on the Earth's surface of 16.3°C according to the Stefan-Boltzmann law. At the depth of the neutral layer, exogenous and endogenous heat fluxes are composed of opposite gradients. The Earth's neutral layer is characterized by a constant temperature throughout the day and year. Within Ukraine, the temperature of the neutral layer varies from 7 to 12°C. Below the neutral layer, the temperature naturally increases with depth. Above the neutral layer temperature changes during the day and year is more affected by exogenous heat, that is, by the Sun's heat than by endogenous heat. Conductive cooling of the Earth at the depth of the neutral layer is almost stopped. The Earth is in an adiabatic thermodynamic state. The surface temperature of the lithosphere remains unchanged at an average of 10°C (283.15 K) with the corresponding heat flux density of 364.46 W/m^2. Failure to account for the close thermal connection between the mantle and the Earth's atmosphere results in erroneous models of the Earth's thermal field, which in turn makes it impossible to create reliable forecast systems.

It should be noted that on the eve of earthquakes in the atmosphere, an increase in the luminescence (up to 30%) of the lower atmosphere, variations of the magnetic and electrical components of the field of noise low-frequency radiation and other changes in the atmosphere of the Earth, etc. are recorded. It is proved that during the earthquake, when the ionization in the D-region of the atmosphere sharply increases, the thermal balance of the upper atmosphere at altitudes of 80-130 km changes. The heating of the upper atmosphere by the flow of energy electrons is related to the action of seismic low-frequency radiation and to the oscillation of the power geomagnetism under the action of moving tectonic plates [7].

The GTME study is performed by thermodynamic and energy analysis methods using the probability deterministic function, which will be more detailed explained in the following paper to this article. Energy analysis method exploits the idea of compatible action at every point in the physical space of laws: the conservation, modification, transfer, and packing of energy in the stochastic movement of physical points.

It is important to note that the thermal balance on the surfaces of the planets of the Solar system, where the amount of endogenous heat generated by the planets, depending on their mass, increases. All planets (except Jupiter and Saturn), at some depth, are in an adiabatic thermodynamic state and have close exogenous and endogenous heat fluxes. Jupiter and Saturn have endogenous heat fluxes greater than exogenous ones. This raises the question of the time profile of the temperature on the Earth's surface from its formation, when the temperature of the entire planet was equal to, say, 6000°C (6273°K), which forms the initial density of the endogenous heat flux by the Stefan-Boltzmann law $8.78*10^7$ W/m^2 for the cold Earth model. For the hot Earth model, the density of

endogenous heat flux is assumed to be constant at 364.46 W/m². Then, the process of the Earth's deposit take place in an adiabatic state with the increase of the internal temperature in the nucleus to a given level of its stabilization.

It is proved that the internal energy of rocks at each depth is in thermodynamic equilibrium with gravitational energy. The rock temperature at each point in the vertical direction is determined by the Stefan-Boltzmann law, which apparently forms the heat exchange determined by the Fourier heat conduction law.

Based on the heat flux densities according to the Fourier law of thermal conductivity and the Stefan-Boltzmann law in the geological environment, the differential equation of temperature distribution by depth for a stationary thermodynamic state of appearance is valid:

$$\lambda_f \cdot grad\left(T_3\right) = z_0 \cdot \alpha_R \varepsilon \sigma \left[\left\{T_3 + z_0 \cdot grad\left(T_3\right)\right\}^4 - T_3^4\right] \quad (1),$$

or
$$\tilde{T}^4(z) - a\tilde{T}(z) + b = 0. \quad (2)$$

Where $a = \dfrac{\lambda_f}{z_0^2 \cdot \alpha_R \varepsilon \sigma}$, $b = \left(\dfrac{\lambda_f}{z_0^2 \cdot \alpha_R \varepsilon \sigma} \cdot T_3 - T_3^4\right)$ are the coefficients of layers of geological medium (GM) at a given depth, which vary in depth and composition of rocks;

$$\tilde{T}(z) = \left[z_0 \cdot \frac{dT_3(z)}{dx} + T_3(z)\right] - \text{GM temperature, °K, which}$$

depends on two main factors, namely, the in-depth compression and absorption of part of the energy of electromagnetic waves emitted by the EIHS from the bowels according to the Stefan-Boltzmann law at a temperature in the center of the nucleus 6273°K; $\sigma = 5,67 \cdot 10^{-8}$ W/m2/°K⁴ is a Stefan-Boltzmann constant.

On the surface of the lithosphere, the energy of the EIHS waves have the density of 364.46 W/m², which, according to the Stefan-Boltzmann law forms a stationary temperature of $T_{30}=283°K$, (273+10) °K. With increasing a depth, the temperature of the rocks increases, that is, the temperature of the Earth is a function of the density of gravitational energy, where E_G - the gravitational energy of the Earth, which increases towards the center.

Performed by Karpenko V.M. and Starodub Yu.P. experimental and theoretical geophysical studies of temperature dependence and stress of rocks in their depth allowed us to develop the T_3 function .

Using the Debye Solid State Theory, which determines the substance density of thermal energy of a solid and the energy approach for modeling the change in the modulus of substance elasticity in GM from the change in gravitational energy. The relation between the density of elastic energy, which depends on its own gravitational energy with the density of thermal energy in the Earth's lithosphere is described by the equation

$$e(T) = e(E_G) \to \rho \frac{3\pi^4}{5\mu} \cdot \frac{RT_3^4}{T_D^3} = \rho V_p^2(E_G)\,\text{J/m}^3, \quad (3)$$

or for the temperature-dependent gravitational energy in the form

$$T_3(E_G) = \sqrt[4]{\frac{5\mu}{3R\pi^4}\left(\frac{\hbar}{k_B}\frac{2\pi}{\lambda_{min}}\right)^3 V_p^5(E_G)}, \text{°K}, \quad (4)$$

where $R = 8,31451$, J/mol/°K is the universal gas constant; $\hbar = 6,626 \cdot 10^{-34}$ J*s is a Planck constant; $k_B = 1,380648 \cdot 10^{-23}$ J/°K is a Boltzmann constant; μ_3, ρ - molar mass and density of matter of the planet, respectively, kg/mol, kg/m³; $T(E_G)$, $T_D = \hbar\Omega_{max}/k$ - internal temperature and Debye matter of planet matter, $\Omega_{max} = 2\pi V_p(E_G)/\lambda_{min}$ °K - the maximum frequency of oscillation of elementary objects of matter of the planet, depending on its gravitational energy, s⁻¹; $V_P(E_G) = \sqrt{V_{P0}^2\left[1 + \ln\left\{1 + E_G(z)/w_0 e_g(z)\right\}\right]}$ - propagation velocity of P-wave at depth z in GM, m/s; V_{P0} - the average velocity of the P-wave on the planet's surface, m/s; $e_g(z) = \rho(z)V_P^2(z)$ - volumetric density of elastic energy, J/m³; w_0 - unit volume of GS, m³; $E_G(z) = \gamma \displaystyle\int_R^{R-z_m} m(z)/(R-z)\,dm(z)$ - the gravitational energy of the planet, J.; V_{s-in}, V_{p-in} - interval velocities of S and P waves, respectively, m/s; $\lambda_{min} = \dfrac{\hbar V_{s-in} V_{p-in}}{T_D k_B}\sqrt[3]{\dfrac{12\pi^2}{\left(V_{s-in}^3 + V_{p-in}^3\right)}}$ - minimum acoustic wavelength, m.

The connection of radiation and conductive heat fluxes of the Earth is also proved, the total thermal conductive power in the atmosphere of solar rays is determined, the temperature of their photons, and the temperature characteristics of the Earth's surface are calculated taking into account the thermal energy of the Sun.

The results obtained provided a sound scientific basis for the development of an Earth model with exogenous and endogenous radiate heat fluxes.

Geothermal model of the Earth for sequential processes of heating and cooling of its phases (inner core, outer core and mantle, crust, atmosphere) is presented in the form for the equations following subsequently for the internal core, external core, crust, and atmosphere:

$$\begin{cases} N_{3-F}dt = C_G d\tau_G + A_G\tau_G dt \ ; \\ A_G\tau_G dt = C_M d\tau_M + A_M\tau_M dt \ ; \\ A_M\tau_M dt = C_K d\tau_K + A_K\tau_K dt \ ; \\ \left(A_K\tau_K + N_{C-3}\right)dt = C_A d\tau_A + A_A\tau_A dt \ ; \end{cases} \quad (5)$$

where $N_{3-F}dt$ - total thermal energy of the Earth, J.; $N_{C-3}dt$ - the total thermal energy of electromagnetic waves from the Sun on the Earth's surface, J.; $C_G d\tau_G$ - the energy of heating the inner core of the Earth, J.; τ_G - the current overage temperature of the inner core above the magma temperature,

°K; $A_G \tau_G dt$ - cooling energy of the surface of the inner core, J.; $C_M d\tau_M$ - heating energy of magma, J.; τ_M - the current excess temperature of magma on its surface above the temperature of the crust, ° K; $A_M \tau_M dt$ - energy of cooling by the magmatic phase for heating the crust and atmosphere, J.; $C_K d\tau_K$ - crust heating energy, J.; τ_K - the current excess crust temperature on the Earth's surface above atmospheric temperature, °K; $\left(A_K \tau_K + N_C\right)dt$ - energy that warms the atmosphere from Earth and the Sun, J.; $A_K \tau_K dt$ - cooling energy of the crust, J.; $C_A d\tau_A$ - atmospheric heating energy, J.; τ_A - the current excess temperature of the atmosphere on the Earth's surface above the temperature of the open physical space, °K; $A_A \tau_A dt$ - energy released by the atmosphere into the open physical space, J.; $A_G = N_{3-F}/T_{G0}$ - coefficient of heat transfer of the surface of the inner core of the Earth, J.°C/s; $A_M = N_3/\overline{T}_M$ - coefficient of heat transfer of the surface of the mantle of the Earth, J./°C/s; $A_K = N_{3-F}/\overline{T}_K$ - heat transfer coefficient of the surface of the Earth's crust, J./°C/s; $A_A = N_{3-F}/\overline{T}_A$ - coefficient of heat transfer of the atmosphere, J./°C/s; $C_G = c_G m_G + c_M m_M + c_K m_K + c_A m_A$ - entropy of all phase states of the substance of the Earth, J./°C; $C_M = c_M m_M + c_K m_K + c_A m_A$ - entropy of magma, crust, Earth's atmosphere, J./°C; $C_K = c_K m_K + c_A m_A$ - entropy of the crust and atmosphere, J./°C; $C_A = c_A m_A$ - entropy of the Earth's atmosphere, J./°C; $T_{G0} = 6000$ - surface temperature of the inner core, °C; $\overline{T}_K = 0,5\left(T_K + T_{K0}\right) = 0,5 \cdot (1320 + 10) = 665$ - the average temperature of the crust between the temperatures of the inner and outer surfaces, °C; $\overline{T}_M = 0,5 \cdot \left(T_я + T_M\right) = 0,5 \cdot (1320 + 4026) = 2673$ - average temperature of the magmatic phase, ° C; $\overline{T}_A = 14,2$ - average temperature of the Earth's atmosphere, °C; $E_3 = N_{3-F} \cdot t_3$ - the total thermal energy spent on heating the Earth during its geological existence during time t_3, J.; $m_G = \rho_G \cdot \frac{4}{3}\pi R_G^3$ - mass of the inner core, kg; $m_M = \rho_M \cdot \frac{4}{3}\pi\left[\left(R_3 - \Delta L_K\right)^3 - R_G^3\right]$ - mass of outer core and mantle, kg; $m_K = \rho_K \cdot \frac{4}{3}\pi\left[\left(R_3\right)^3 - \left(R_3 - \Delta L_K\right)^3\right]$ - mass of the crust, kg; $m_A = 5,3 \cdot 10^{18}$ - given mass of the atmosphere, kg; $\rho_G, \rho_M, \rho_K, \rho_A$ - average densities of the nucleus, magma, crust, Earth's atmosphere, respectively, kg/m³; $N_{3-F} = A_K \tau_K = \eta_{3-f} \cdot S_3 = \eta_{3-f} \cdot 4\pi R_3^2$ - set the total thermal power of the conductive heating-cooling on the surface of the Earth's crust, W; $\eta_{3-f} = 0,087$ - the average density of thermal

conductive flow on the surface of the crust, W/m². The first equation describes the thermal processes in the Earth's inner core, the second - in the outer core and mantle, the third - in the Earth's crust, and the fourth - in the atmosphere. Only a consistent consideration of all these relations can ensure the construction of a reliable geothermal model of the Earth.

Analysis of the developed geothermal model of the Earth with heat fluxes indicates an increase in the planet's temperature with constant total gravitational energy with increasing depth, as well as the existence of the following physical sequential mechanisms that are triggered by the thermal energy when approaching the Earth's center with an increase in the thermal energy density.

The new geothermal model of the Earth provides the scientific basis for a more accurate prediction of the geodynamic activity of the Earth, which will help to solve the important task of civil protection - ensuring the safety and protection of the population and territories from the negative consequences of emergencies.

V. CONCLUSIONS

The developed geothermal model of the Earth forms a new understanding of the thermal processes of our planet and is the basis for the creation of the tools for predicting dangerous natural processes of endogenous origin, the development of new energy saving geotechnologies.

The scientific significance of the geothermal model of the Earth with exogenous and endogenous heat fluxes substantiate the existence of an internal source of thermal energy in the center of the Earth's core, the energy is absorbed in a small part by the substance of the Earth, and the larger part is emitted into open physical space. The endogenous source is caused by the change of thermoplastic parameters of the substance with the change of depth, which are characterized by temperature gradients, conductive heat fluxes and velocities of acoustic waves.

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