

Запропоновано підхід до синтезу контурів систем автоматичного керування на основі дробового характеристичного поліноmu, який дозволяє забезпечувати бажану якість перехідного процесу за умови реалізації певної структури дробового регулятора, яка залежить від передавальної функції об'єкта керування. Застосування дробових бажаних форм розширює гамму можливих налаштувань регуляторів дробового порядку при синтезі контурів електромеханічних систем, забезпечує кращу якість перехідних процесів, порівняно з регуляторами цілого порядку, і тим самим підвищує ефективність синтезованих систем. На основі отриманих результатів досліджень для налагодження контурів електромеханічних систем можна рекомендувати запропоновані дробові бажані форми, як такі, що задовольняють бажані вимоги до систем керування електромеханічних систем. Побудова електромеханічних систем за принципом керування з послідовною корекцією має суттєву перевагу над іншими системами, завдяки простоті налаштування кожного з контурів, а також можливості реалізації обмеження координат регулювання. Розроблена методика структурно-параметричного синтезу регуляторів дробового порядку за умови їх каскадного ввімкнення в багатоконтурних електромеханічних системах та наведено алгоритм синтезу регуляторів дробового порядку відповідних контурів регулювання. Проведено синтез електромеханічної системи з каскадним ввімкненням регуляторів шляхом застосування удосконаленого методу узагальненого характеристичного полінома для вибору структури і параметрів регуляторів дробового порядку та застосування бажаної форми дробового порядку. Як приклад, розглянуто двоконтурну систему підпорядкованого регулювання, де об'єктом регулювання є електропривод «тиристорний перетворювач – двигун». Досліджено вплив синтезованих регуляторів дробового порядку на динамічні властивості електромеханічної системи «тиристорний перетворювач – двигун». Проведені дослідження показали можливість реалізації каскадно ввімкнених регуляторів для електромеханічних систем, де поєднуються контури з передавальними функціями цілого і дробового порядку, а також систем з контурами тільки дробового порядку

Ключові слова: електромеханічна система, регулятори дробового порядку, синтез, передавальні функції дробового порядку

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1. Introduction

As regards the electromechanical systems (EMS), one most often faces the structural-parametric methods of synthesis. A feature of such systems is:

– along with splitting the system into two parts, namely: a control object and a correction link (controller), it is very often that the inertia of some functional parts in the correc-

tion links can be disregarded in comparison with the inertia of the control object itself;

– the use of standard controllers.

Therefore, a common task for the synthesizing EMS is reduced to the choice of a standard controller that could ensure the assigned control law.

Significant progress in the design of EMS was associated with the transition to the synthesis of optimal systems of subor-

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DEVISING A PROCEDURE FOR THE SYNTHESIS OF ELECTROMECHANICAL SYSTEMS WITH CASCADE-ENABLED FRACTIONAL-ORDER CONTROLLERS AND THEIR STUDY

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dinate regulation (SSR), proposed by Kesler. Such systems implement the principle of consistent correction by the cascading switching of controllers for each regulation coordinate [1, 2].

Each circuit within such a system is reduced to a second-order link, which is described by the following characteristic polynomial:

$$H(s) = T^2 s^2 + 2\xi Ts + 1.$$

If the damping coefficient is $\xi = \frac{\sqrt{2}}{2} = 0.707$, then the dynamic characteristics are the following: the magnitude of overshooting $\sigma = 4.33\%$; the time the steady value is first achieved $t_n = 4.7T_\mu$; control time $t_p = 8.4T_\mu$ (T_μ is the decompensated time constant of the control loop of the corresponding coordinate). Such characteristics of the dynamic process fall under a technically optimal class and they are, in addition to the title Betragsoptimal, designated as a “modular optimum”. Controllers are synthesized under condition for achieving such a control criterion for each coordinate. If one changes the SSR contours optimization criterion in accordance with some standard form of the full order, we obtain controllers in each contour [3, 4], which would provide for the required dynamic characteristics according to the selected form of root distribution in a characteristic equation in the complex plane.

The construction of EMS on the principle of control with sequential correction has a significant advantage over other systems, owing to the simplicity of adjusting each contour, as well as a possibility to implement the regulation coordinate constraints.

Studies show that the use of fractional-order controllers in EMS can improve the quality of transients. In this case, it is possible to increase the margin of stability compared to similar systems, which use classic (full-order) controllers. Application of fractional-order controllers in order to manage production processes, instead of full-order PID controllers, will be intensive if we solve scientific and applied problems related to the synthesis and implementation of such systems, as well as methods to adjust them. At present, however, the issue of synthesis and implementation of the cascade enabled fractional-order controllers, as well as studying their capabilities in automated EMS, needs to be resolved.

Thus, the relevance of research in this field is explained by the need to devise methods to synthesize SSR that are described by fractional-order transfer functions. Such systems emerge if a control object is described by a fractional order, or the desired standard form corresponds to the characteristic polynomial of fractional order [5, 6].

2. Literature review and problem statement

The results of our analysis of the recent literary sources reveal that application of fractional calculus is one of the most promising approaches to the development and study of modern EMS. The use of the desired fractional forms during the synthesis of single-circuit control systems is considered in paper [5]. For multi-circuit systems, the synthesis procedure was not considered. Study [6] considers the application of the desired fractional forms to expand the range of possible adjustments for fractional-order controllers in the synthesis of electromechanical systems; it also proves that the fractional-order controllers provide the best quality of transitional processes compared to the full-order controllers. Experimen-

tal testing of the fractional-order controllers, synthesized by the proposed approaches, performed in the system of speed control of the system “frequency converter – induction motor”, confirmed efficiency in terms of extending control possibilities of a fractional controller in comparison with a classic PID-controller. However, the cited study addressed only single-contour systems with a single feedback. In the cited work, similar to the preceding one, the synthesis procedure is limited to a single circuit.

Papers [7, 8] demonstrate the advantages of using fractional PI^λ -controllers in a closed system of synchronous engine control with permanent magnets, which helped obtain certain dynamic characteristics and ensure the system robustness. However, the variant of synthesis, considered in [7], is not uniform for different fractional-order circuits. It should be noted that publication [8] applied an evolutionary algorithm for the synthesis of fractional controllers, but failed to devise an engineering procedure for the synthesis of multi-circuit EMS of fractional order. It was demonstrated in [9, 10] that in the closed control systems over the speed and position of DC motors (DCM) the use of PI^λ -fractional-order controllers is more effective compared to classic PI-controllers when configuring circuits for a symmetrical optimum. However, the cited studies apply to single-circuit systems only. The authors of work [11], when creating a new system for controlling an induction motor for an electric vehicle, used a PI^λ -controller draw a conclusion on that the properly designed and implemented PID^μ -controller is better than a classic PID-controller. The same conclusion was drawn in [12], whose authors investigated a PID^μ -fractional-order controller for a system that controls position of an SSR-based mechanism considering the saturation of the magnetic system and the shaft elasticity. However, the cited studies addressed only single-contour systems with a single controller.

Any synthesis of EMS controllers implies the criterion of system optimization. The optimization criteria for integer controllers have been sufficiently developed in the form of standard types of characteristic polynomials. The standard (desirable) fractional-order forms, similarly proposed in [5, 6], are used for the synthesis of fractional EMS.

Paper [13] performed a synthesis of the fractional-order PI and PD-controllers to control the speed and position of the DC motor shaft with parallel excitation. It should be noted that the criteria for optimizing the synthesis of such systems were not considered. In addition, the cited paper contains no unified approach to the synthesis of controllers with different contours. The results of experimental research have shown that fractional-order controllers are superior to classical controllers in terms of dynamic EMS characteristics. It should be noted that the disadvantage of the cited paper is that the authors synthesized only the outer controllers of speed and position based on a general transmission function of the system without studying and synthesis of the internal current contour. The same drawback is noticed in work [14]. In addition, in it, the synthesis of controllers employed a common transfer function of the system, for which the input signal is the supply voltage of a DCM armature, and the output signal is the shaft rotation frequency. Such an approach does not take into consideration the dynamic characteristics of a thyristor transducer.

Papers [15, 16] compared different methods of evolutionary optimization for adjusting fractional- and integer-order controllers in a DCM speed control system. Three procedures for the synthesis of controllers were considered: a genetic algorithm, particle-swarm optimization, and a dif-

ferential evolution; the result is a comparison based on the quality indicators and robustness of the designed system. However, these papers are also characterized by the same shortcomings related to the lack of synthesis of the internal contour and, therefore, the impossibility to implement a current constraint necessary to ensure the appropriate dynamic characteristics.

Thus, the unresolved task in the examined literary sources is the synthesis of multi-circuit electromechanical systems with the cascade (subordinate) enabled controllers of fractional order and ensuring the desired dynamic characteristics of each adjustment contour. To solve this issue, it is necessary to devise procedures for the synthesis of fractional-order controllers that could be cascade enabled for all circuits of SSR control, similarly to the full-order EMS. That would make it possible to devise an engineering procedure for the synthesis of cascade enabled fractional controllers for multi-contour EMS.

3. The aim and objectives of the study

The aim of this study is to devise a procedure for the structural-parametric synthesis of fractional-order controllers under condition that they are cascade enabled in multi-circuit electromechanical systems. This makes it possible to create new and upgrade existing electromechanical systems with cascade-enabled controllers with an extended spectrum of dynamic properties, which correspond to the desired forms of fractional order. In addition, a procedure for the construction of automatic control systems for a fractional-order object of control is implemented.

To achieve the set aim, the following tasks have been solved:

- to analyze the procedure for the synthesis of systems described by fractional-order TF;
- to modernize the method of synthesizing a generalized characteristic polynomial in relation to electromechanical systems of fractional order;
- to synthesize the controllers of armature current and the speed of a DC motor, cascade-enabled, and to investigate the dynamic characteristics of these electromechanical systems;
- to investigate a static error due to the effect of disturbance in the synthesized electromechanical systems.

4. Application of the method of a characteristic fractional polynomial for the structural-parametric synthesis of fractional-order controllers

4.1. Analysis of the procedure for synthesizing fractional-order systems and modernization of the method for synthesizing a generalized characteristic polynomial for such systems

EMS may include those in which control objects are described by the fractional-order characteristic polynomials q [17–20]. In this case, as well as in the case where a control system implies using a fractional controller, the synthe-

sis of an EMS of fractional order can be approached similarly to the procedure for full-order SSR. In this case, it should be borne in mind that the optimization criterion would correspond to the desired characteristic polynomial $H_d(s)$ of fractional order. For it, the transition functions of control coordinate are known, so the choice of $H_d(s)$ is based on the requirements to the dynamic characteristics of a given control coordinate.

It is proposed to improve the method of a generalized characteristic polynomial (GCP) [3, 4] in order to select the structure and parameters of fractional-order controllers. One must first select some desired form of fractional order. The following two forms are proposed in paper [3]: No. 1 and No. 2. The expressions of their transfer functions (TF) are as follows:

$$W_{s1}(s) = \frac{\omega_{o1} / K_1}{s^q + \omega_{o1}}, \tag{1}$$

$$W_{s2}(s) = \frac{\omega_{o2}^q / K_1}{(s + \omega_{o2})^q}, \tag{2}$$

where ω_{o1} , ω_{o2} is the average geometric root of the corresponding desired form, which determines the system performance, K_1 is the gain factor of a feedback for the control coordinate, q is the fractional order of a characteristic polynomial.

The algorithm of a GCP method for the synthesis of fractional-order controllers is as follows:

1. According to the specified structural scheme of a closed circuit, define its TF.
2. By dividing the numerator and denominator of the derived TF by the numerator, we obtain an expression that resembles the desired fractional form by its structure.
3. Choose, as desired, fractional form No. 1 (1) or form No. 2 (2). Based on the desired parameters for the transition process: $(\delta, t_{0,95})$, the requirement is set for the transformation of the expression derived in the previous point of a given algorithm into the TF expression of the desired form with the help of implementing the appropriate TF of the controller.
4. From the condition for the identity of characteristic polynomials of the control circuit and based on the chosen form, we derive the equation from which the fractional controller's TF is synthesized.

As an example, we shall consider a dual-circuit SSR where the object of control is the electric drive “thyristor transducer (converter) – engine (motor)” (TT-E). The structural scheme of such an EMS is shown in Fig. 1.

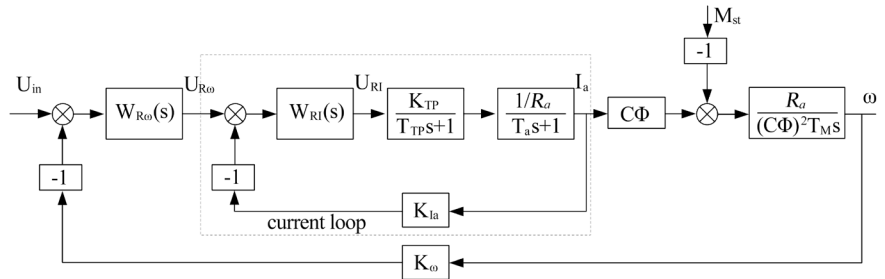


Fig. 1. Structural scheme of a dual-circuit engine speed SSR ($W_R(s)$ – transfer function of the control circuit for armature current I_a ; $W_{Ro}(s)$ – transfer function of the control circuit for engine angular velocity ω ; K_{Ia} – gain factor of current sensor, K_{ω} – gain factor of velocity sensor, K_{TP} – gain factor of TF; T_{TP} – time constant of TF; R_a – total resistance of armature circuit; C – engine structural constant; Φ – excitation magnetic stream; T_a – electromagnetic time constant of the armature circuit; T_M – electromechanic time constant time of electric drive; M_{st} – static momentum of loading)

4. 2. Synthesis of controller for the current of a DC motor armature using the desired forms of the full and fractional order

The generally accepted sequence in the synthesis of SSR with cascade-enabled controllers is the synthesis of controllers, starting from the inner circuit, proceeding to the outer circuit, where there is a controller that is synthesized after the synthesis of the inner contour is completed. Therefore, we shall first consider the circuit that controls the armature current provided that the internal feedback for the engine's e.m.f. can be ignored.

According to the structural scheme shown in Fig. 1, TF of the closed current circuit $W_I(s)$ takes the form

$$W_I(s) = \frac{W_{RI}(s) \frac{K_{TP}}{(T_{TP}s+1)} \frac{1/R_a}{(T_a s+1)}}{1 + W_{RI}(s) \frac{K_{TP}}{(T_{TP}s+1)} \frac{1/R_a}{(T_a s+1)} K_{Ia}} \tag{3}$$

By dividing the numerator and denominator of the derived TF by the numerator, and considering that the expression for the desired characteristic polynomial is of the first order, we obtain

$$W_I(s) = \frac{1}{\frac{(T_{TP}s+1)(T_a s+1)R_a}{W_{RI}(s)K_{TP}s} + K_{Ia}} \tag{4}$$

Let us assign, as the desired integer standard form, for example, a binomial or a first-order Butterworth with TF

$$W_{sl}(s) = \frac{1/K_{Ia}}{T_{\mu l}s+1} \tag{5}$$

where $T_{\mu l}$ is the so-called small non-compensated time constant of the circuit that adjusts the armature current, which takes into consideration T_{TP} , as well as inertia of the armature current measurement system and the filter of higher harmonics.

The magnitude $T_{\mu l} < T_a$, which means that the performance of the synthesized current circuit is due only to a small time constant.

Put the requirement for the transformation of expression (4) into expression (5).

From the condition for identity of characteristic polynomials $H_{sl}(s)$ and $H_I(s)$, we obtain:

$$\frac{(T_{\mu l}s+1)(T_a s+1)R_a}{2W_{RI}(s)K_{TP}K_{Ia}T_{\mu l}s} = 1 \tag{6}$$

Next, find

$$W_{RI}(s) = \frac{(T_{\mu l}s+1)(T_a s+1)R_a}{2K_{TP}K_{Ia}T_{\mu l}s} \tag{7}$$

Substituting the parameters for links included in the current circuit, we obtain a current controller's TF.

$$W_{RI}(s) = 1.22 + \frac{22.896}{s} + 0.003778s.$$

That is, we have derived the integer PID-controller for the integer standard form. Fig. 2 shows a transfer function of the armature current as the response from the inner current circuit to the effect of a jump-like setting influence for this circuit.

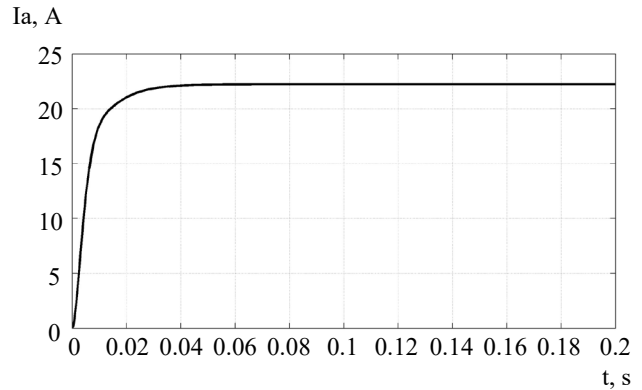


Fig. 2. Transition function of the optimized circuit with the integer PID current controller

Simulation study for a given case, as well as for all the following systems, involved stimulation models implemented in the software package MATLAB Simulink and NINTEGER, an application to the MATLAB Simulink package.

Similarly, we synthesize the current controller, based on the desired fractional form No. 1 $W_{s1}(s)$ (1).

Put the requirement for the transformation of expression (3) into expression (1), which was introduced with a feedback coefficient $K_1 = K_{Ia}$.

From the condition for identity of $W_I(s)$ and $W_{s1}(s)$, we obtain:

$$\frac{1}{\frac{(T_{\mu l}s+1)(T_a s+1)R_a}{W_{RI}(s)K_{TP}} + K_{Ia}} = \frac{\omega_{oc}/K_{Ia}}{s^q + \omega_{o1}} \tag{8}$$

By solving equation (8), we obtain:

$$W_{RI}(s) = \frac{(T_{\mu l}s+1)(T_a s+1)R_a \omega_{o1}}{K_{TP}K_{Ia}s^q} \tag{9}$$

Thus, the result of synthesis is the derived structure of an armature current controller of fractional order.

Let the desired dynamic characteristics for an armature current be the following: overshooting $\delta = 6.80\%$ and time to reach 0.95 of the stable value $t_{0.95} = 0.042$ s. This is ensured by the desired form (line No. 5, Table 1) with parameters $q = 1, 2, \omega_{o1} = 100 \text{ s}^{-1}$.

Table 1
Parameters for transitional functions of the desired form of fractional order No. 1 for $\omega_{o1} = 100 \text{ s}^{-1}$

No.	q	$\delta, \%$	t, s	T_{reg}, s
1	0.8	0	0.0283	0.0283
2	0.9	0	0.0299	0.0299
3	1.0	0	0.0319	0.0319
4	1.1	2.5	0.0361	0.0361
5	1.2	6.76	0.0424	0.1106
6	1.3	11.52	0.0506	0.1628

By substituting parameters of the links included in the current circuit, we obtain a current controller's TF for $\omega_{o1} = 100 \text{ s}^{-1}$.

$$W_{RI}(s) = \frac{0.805}{s^{0.2}} + \frac{15.111}{s^{1.2}} + 0.0025 s^{0.8} \tag{10}$$

By using such a controller, we obtain a transition process with the following parameters: $\delta=7.0\%$, $t_{0.95}=0.0404\text{ s}$ (Fig. 3, curve 5), that is the deviation from the specified parameters does not exceed 1%.

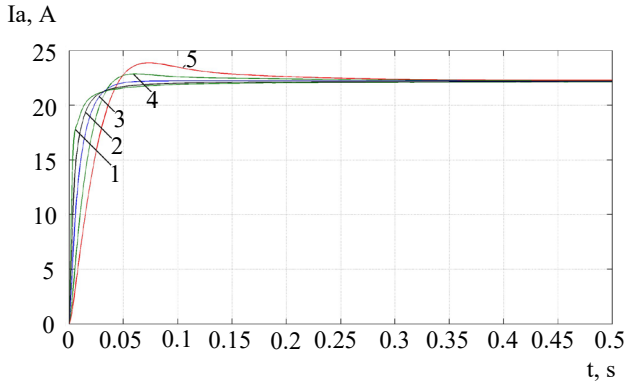


Fig. 3. Transition functions of the optimized current circuit with a current controller (9) if: $q=0.8$ – curve 1; $q=0.9$ – curve 2; $q=1.0$ – curve 3; $q=1.1$ – curve 4; $q=1.2$ – curve 5

By substituting EMS parameters for condition ($q=0.8-1.1$ for $\omega_{o1}=100\text{ s}^{-1}$), we derive the following expressions for a current controller's TF

$$W_{RI}(s) = 0.805s^{0.2} + \frac{15.111}{s^{0.8}} + 0.0025s^{1.2} \text{ for } q=0.8,$$

$$W_{RI}(s) = 0.805s^{0.1} + \frac{15.111}{s^{0.9}} + 0.0025s^{1.1} \text{ for } q=0.9,$$

$$W_{RI}(s) = 0.805 + \frac{15.111}{s} + 0.0025s \text{ for } q=1.0,$$

$$W_{RI}(s) = \frac{0.805}{s^{0.1}} + \frac{15.111}{s^{1.1}} + 0.0025s^{0.9} \text{ for } q=1.1.$$

By examining the transition functions of an armature current under condition of change in q in the range from 1.0 to 1.2 for $\omega_{o1}=100\text{ s}^{-1}$, we obtain a family of graphs shown in Fig. 3.

Such an approach makes it possible to synthesize a current controller also under condition of using another desired form of fractional order.

4.3. Synthesis of a direct current engine speed controller using the desired fractional-order forms

Next, consider a procedure for synthesizing a controller of the engine angular velocity taking into consideration that the internal current circuit is optimized, either for the condition of the desired shape of the full or fractional order. At the same time, the optimization criterion for a velocity circuit will correspond to the desired fractional form.

First, consider the case of optimizing a velocity circuit under condition that the current circuit is synthesized according to (5), that is, the integer TF. Then the transfer function of the closed velocity circuit, taking into consideration the TF of the optimized current circuit (5), takes the form:

$$W_{\omega}(s) = \frac{W_{R\omega}(s) \frac{1/K_{Ia} R_a}{T_{\mu l} s + 1} C\Phi T_M s}{1 + W_{R\omega}(p) \frac{1/K_{Ia} R_a}{T_{\mu l} s + 1} C\Phi T_M s K_{\omega}} \quad (11)$$

By dividing the numerator and denominator of the derived TF by the numerator, we obtain

$$W_{\omega}(s) = \frac{1}{\frac{K_{Ia} (T_{\mu l} s + 1) C\Phi T_M s}{W_{R\omega}(s) R_a} + K_{\omega}} \quad (12)$$

Choose, as the desired, a fractional form (1). Then we can record for the velocity circuit:

$$W_{s1}(s) = \frac{\omega_{o\omega} / K_{\omega}}{s^q + \omega_{o\omega}} \quad (13)$$

where $\omega_{o\omega}$ is the desired value of the mean geometric root of the velocity circuit.

From the equality condition $W_{\omega}(s) = W_{s1}(s)$, we obtain the following expression for the velocity controller's TF

$$W_{R\omega}(s) = \frac{(T_{\mu l} s + 1) C\Phi T_M K_{Ia} \omega_{\omega} s}{R_a K_{\omega} s^q} \quad (14)$$

By substituting EMS parameters under condition ($q=0.8-1.2$ for $\omega_{o\omega}=10\text{ s}^{-1}$), we derived the following velocity controller's TFs:

$$W_{R\omega}(s) = 8.8s^{0.2} + 0.0581s^{1.2} \text{ for } q=0.8,$$

$$W_{R\omega}(s) = 8.8s^{0.1} + 0.0581s^{1.1} \text{ for } q=0.9,$$

$$W_{R\omega}(s) = 8.8 + 0.0581s \text{ for } q=1.0,$$

$$W_{R\omega}(s) = \frac{8.8}{s^{0.1}} + 0.0581s^{0.9} \text{ for } q=1.1,$$

$$W_{R\omega}(s) = \frac{8.8}{s^{0.2}} + 0.0581s^{0.8} \text{ for } q=1.2.$$

Table 2

Parameters for transition functions of the desired form of fractional order No. 1 for $\omega_{o1}=10\text{ s}^{-1}$

No.	q	$\delta, \%$	$t, \text{ s}$	$t_{reg}, \text{ s}$
1	0.8	0	0.485	0.485
2	0.9	0	0.365	0.365
3	1.0	0	0.3	0.3
4	1.1	2.7	0.28	0.28
5	1.2	7.3	0.28	0.75
6	1.3	13.3	0.29	0.94

By examining the transition functions of engine velocity, we obtain a family of graphs shown in Fig. 4.

Let us now consider the case of optimizing a velocity circuit under condition that the current circuit was synthesized according to (1), that is, there is a fractional TF of the current circuit.

In this case, we can write the following expression for the TF of a velocity circuit:

$$W_{\omega}(s) = \frac{W_{R\omega}(s) \frac{\omega_{o1} / K_{Ia} R_a}{s^q + \omega_{o1}} C\Phi T_M s}{1 + W_{R\omega}(s) \frac{\omega_{o1} / K_{Ia} R_a}{s^q + \omega_{o1}} C\Phi T_M s K_{\omega}} \quad (15)$$

By dividing the numerator and denominator of the derived TF by the numerator, we obtain

$$W_{\omega}(s) = \frac{1}{(s^q + \omega_{ol})C\Phi T_M K_{Ia} s + K_{\omega} W_{R\omega}(s)\omega_{ol}R_a} \quad (16)$$

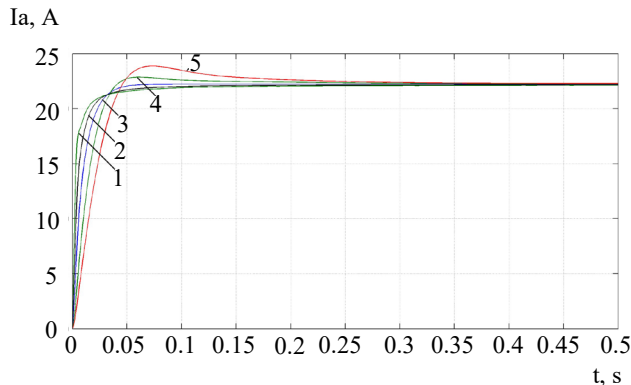


Fig. 4. Transition functions of the optimized velocity circuit with a velocity controller (14) if: $q=0.8$ – curve 1; $q=0.9$ – curve 2; $q=1.0$ – curve 3; $q=1.1$ – curve 4; $q=1.2$ – curve 5

Similar to the previous case, we choose, as the desired, a fractional form (13). Then, from the equality condition $W_{\omega}(s)=W_{s1}(s)$, we obtain:

$$W_{R\omega}(s) = \frac{(s^q + \omega_{ol})C\Phi T_M K_{Ia} \omega_{os} s}{R_a \omega K_{\omega} s^{qsh}} \quad (17)$$

By substituting the EMS parameters under condition ($q=0.8-1.2$ for $\omega_{os}=10 \text{ s}^{-1}$), we derive the following expressions for the TF of a velocity controller

$$W_{R\omega}(s) = 8.8s^{0.2} + 0.088s^{1.2} \text{ for } q=0.8,$$

$$W_{R\omega}(s) = 8.8s^{0.1} + 0.088s^{1.1} \text{ for } q=0.9,$$

$$W_{R\omega}(s) = 8.8 + 0.088s \text{ for } q=1.0,$$

$$W_{R\omega}(s) = \frac{8.8}{s^{0.1}} + 0.088s^{0.9} \text{ for } q=1.1,$$

$$W_{R\omega}(s) = \frac{8.8}{s^{0.2}} + 0.088s^{0.8} \text{ for } q=1.2.$$

The transition processes of velocity in such a synthesized double-circuit SSR are similar to the results shown in Fig. 4. This is explained by that the obtained expressions for the TF of velocity controllers were derived based on the same expressions for the desired fractional TFs.

4. 4. Analysis of static errors due to the effect of disturbance in the synthesized electromechanical systems

Under condition of a load surge in the form of a static load momentum at second 8, there is a static error, which is shown enlarged in the chart in Fig. 5. Its magnitude is in the range: 3.67 % – curve 1; 2.81 % – curve 2; 1.49 % – curve 3; 1.35 % – curve 4; 1.26 % – curve 5.

Thus, the synthesized EMS is static and its static error depends on the value of q .

Somewhat different were the processes to treat a load surge, but only for $q < 1$. This difference manifested itself by a decrease in the static error for curve 1, by 0.03 %, and curve 2 – by 0.02 %, which is disparagingly small. It should be noted that increasing the value of q decreases the static

error due to a disturbance effect. Our study has shown that under condition of $q \geq 1.7$ the system's property of parametric astaticism is almost achieved (a static error from the disturbance is absent). Along with this, there is an increase in the adjustment time relative to the job action, and under condition $q=1.9$ there are weakly damped fluctuations in the engine velocity, under condition of executing both the job and the disturbance. If one accepts $q=2.0$, that is condition (13) is transformed into an integer transfer function of an unstable system, the simulation studies have confirmed this instability.

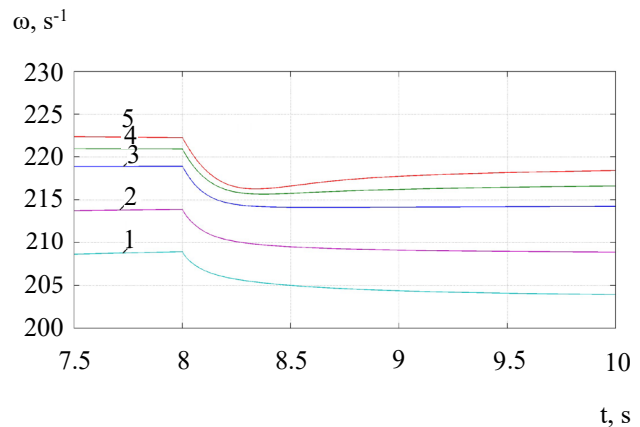


Fig. 5. Transition functions of the optimized velocity circuit with a velocity controller (14) at the time of a load surge, if: $q=0.8$ – curve 1; $q=0.9$ – curve 2; $q=1.0$ – curve 3; $q=1.1$ – curve 4; $q=1.2$ – curve 5

5. Discussion of results from a procedure development

The proposed approach to the synthesis of ACS circuits based on a fractional characteristic polynomial makes it possible to ensure the desired quality of a transition process under condition of implementing a certain structure of the fractional controller, which depends on the transfer function of a control object. The application of fractional desirable forms expands the range of possible fractional-order controllers' configurations in the synthesis of EMS circuits, ensures a better quality of transients compared to the full-order controllers, and thereby improves efficiency of the synthesized systems. Based on the findings from our research, we can recommend, for adjusting the EMS circuits, using fractional desirable forms – form No. 1 (1) at $q=0.9-1.8$, or form No. 2 (2), which could meet the desired requirements to the systems of control over EMSs.

It should be noted that the EMS circuits synthesis with respect to form (2) has its peculiarities. The dynamic characteristics of such systems are also different, in particular, in this case, for $q=2.0$, the system could be robust and would match a second order link. The issue on synthesizing controllers in such systems is that the characteristic polynomial of the desired form (2) needs a preliminary expansion in some series, for example, by Taylor. The number of terms in the series influences the accuracy of approximation and, therefore, requires additional analysis. The purpose of such an analysis is to establish the optimal number of terms, based on the conformity of dynamic characteristics of the desired shape (2) and its approximation. Further research may address solving a given task.

6. Conclusions

1. We have modernized a method for the synthesis of a generalized characteristic polynomial for fractional-order EMS and constructed an algorithm for synthesizing fractional-order controllers for appropriate control circuits. The modernization implies finding the transfer functions of fractional controllers based on building and solving an equation for the desired form of fractional order.

2. The current and velocity controllers have been synthesized, characterized by a wide range of dynamic properties; it was found specifically that an increase in the value of q entails increasing the magnitude of overshooting when executing the job signal and decreasing the static velocity error

in a two-circuit EMS due to the effect of disturbance in the form of a static load momentum.

3. Based on an analysis of the dynamical properties of systems, we have shown a possibility to implement cascade-enabled controllers for EMS, which combine circuits with TF of the integer and fractional order, as well as systems with fractional-order circuits only.

4. For the synthesized EMS, according to form (1), it was established that such systems are characterized by parametric astatism, which is observed for $q > 1.7$. At the same time, it should be borne in mind that, starting from $q = 1.9$, there emerge the weakly damped oscillations of control coordinates, which pass into the nondamping ones at $q = 2.0$, that is the system loses robustness.

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