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Influence of boundary forces on the strength of a thin-walled cylindrical tank

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Abstract. The strength of a thin-walled cylindrical large-capacity tank was evaluated**.** The walls of the tank and the bottom are simultaneously exposed to hydrostatic pressure of the liquid and additionally gas pressure-loaded. The stress at the bottom of the tank was determined taking into account the boundary forces under the combined action of hydraulic and gas pressures. The dependences for determining the boundary forces were obtained by solving the equation of deformations compatibility of a thin-walled cylindrical shell and a round plate resting on a solid surface. The influence of gas pressure increase on the magnitude of stresses was investigated.

1. Introduction and literature review

Large-capacity cylindrical tanks are widely used in the industry for storage of liquids: petroleum, oil, chemicals, etc. Vertically placed steel cylindrical tanks are welded thin-walled structures with a flat round bottom. The walls and bottom of the liquid-filled tanks are exposed to hydrostatic pressure. External action of wind load on the tank wall can lead to buckling collapse if the tank is partially filled with liquid. In case of a shortcoming of valve devices or their failure, an additional gas pressure appears in tanks. The presence of microcracks and other material defects on the inner surfaces of the tanks contributes to the concentration of stresses and reduces the strength of the structure. Therefore, the action of various factors can lead to damaging the tanks filled with liquid. Leakage of liquid from the tank is potentially dangerous to the environment due to pollution, fire or explosion hazard [1]. This causes significant financial losses and the emergence of dangerous situations for humans and the environment.

2. Problem formulation

Today, vertical cylindrical tanks are the most convenient and rather cost-effective device for storing liquids: petroleum, oil, chemicals, etc. Vertical cylindrical tanks with low pressure are designed under the assumption that the excess pressure inside the tank is almost equal to atmospheric pressure ($p = 93.3\ kPa$) [1]. Therefore, the tank wall will only be exposed to the hydrostatic pressure of the liquid. However, in case of deficiency of valve devices or their failure the gas pressure occurs in the tanks, the value of which exceeds the value of atmospheric pressure. Thus, failing to take into account the action of additional pressure on the cylindrical tank walls adversely affects its operational reliability.

In [2] it is noted that 7% of emergencies that occur during the use of tanks are the result of structural defects, and 50% are caused by defects during installation and manufacture of tank structures. Vertical steel tank designs involve a weak weld, connecting the stationary roof to the tank walls. This weld protects the tank from the destruction of its walls when the internal pressure is increased due to the destruction of the roof. In addition, the work [2] emphasized that the world

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practice showed low efficiency of such protection against tank destruction. There were often cases when the roof of the tank did not come off, but the bottom of the tank teared off and the whole cylindrical structure rose into the air.

According to [3], uneven subsidence of the tank structure over the area and along the bottom perimeter cause additional deformations in the elements of the structure, especially in the lower junction of the wall with the edge of the bottom, which causes additional stresses. The combination of working stresses with stresses from uneven subsidence of the bottom can lead to the destruction of the wall-bottom joint.

Therefore, given the relevance of the issue of the tanks' strength in general and to avoid the destruction of the junction of the wall with it, this junction must be calculated taking into account the edge effect.

3. Analysis of publications

The strength of cylindrical, conical and spherical tanks and the influence of various factors on their strength were considered in many works, since today the problems of strength remain relevant. The study [4] covers a number of important issues related to the strength of tanks, taking into account the design features and plastic deformation of the material. The influence of weld joint defects on the strength of tanks was studied using the finite element method in the publication [5]. Stresses in cylindrical vessels under pressure are considered in the publication [6]. In this study, both the theoretical research of the main stresses in the wall of the cylindrical tank and the experimental study of these stresses were conducted. In general, the issue of the strength of tanks with different shapes, the design models of which are cylindrical, spherical and conical shells, is covered in the works [7-10]. The influence on the strength of the tank walls by the existing stress concentrators was investigated in study [11] using the finite elements method. According to [3], the most common tank accidents are brittle fractures. As an example, this paper shows the origin of a crack at the tank's wall-bottom joint, made of low-alloy steel of 09G2S-15 mark. A significant number of published works testify to the relevance of assessing the strength of tanks for various purposes, taking into account their stress.

4. Aim of Paper

The purpose of the work is to study the combined effect of hydraulic and gas pressure on the strength of a steel cylindrical thin-walled tank in the places of walls-bottom joint, taking into account the boundary. The object of the study is the change of stresses in the wall of a cylindrical steel tank, arising from the combined action of hydraulic and gas pressure, taking into account the boundary forces. The subject of the study is a large-capacity cylindrical steel tank, the bottom of which rests on a well-compacted sand cushion.

5. Materials and methods

Materials of structures of large-capacity cylindrical tanks are low-alloy steels for engineering structures of the C255, C275, C285 brands, in the form of sheet or broadband rolled metal [1]. C255 steel analogues in the European Union are S275JR, S275J0 and S275J2 steels. The mechanical characteristics of C255 steel with a sheet thickness of 10-20 *mm* are as follows: yield strength is 245 *MPa*, temporary resistance limit is 370 *MPa*.

The design model of a large-capacity tank is a thin-walled cylindrical shell with a medium radius R and wall thickness δ_1, the bottom of which is in the form of a round plate δ_2 thick resting on a compacted sand cushion (Figure 1). The tank is under the action of hydrostatic pressure q of the liquid with a density ρ that fills the tank to a height H and is additionally loaded with gas pressure p. In the case of such a load on the cylindrical shell, the dangerous cross-sections in which the greatest stresses occur are located near the joints between the wall and the bottom.

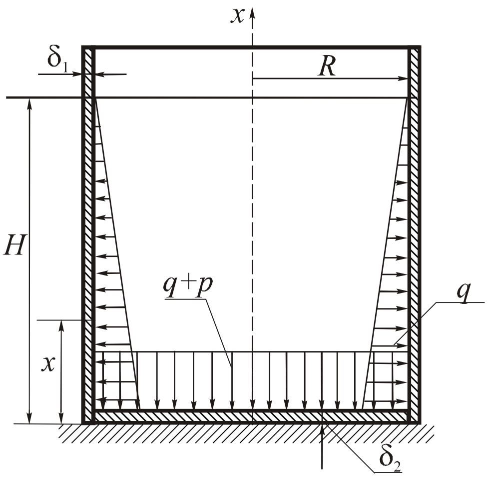

Figure 1. The design model of the cylindrical tank.

The internal forces (Figure 2) in the wall of the cylindrical tank near the wall-bottom joint in accordance with [12] are the circular force $T(x)$, meridian $M(x)$ and circular $K(x)$ bending moments:

$$T(x) = -2kRP_0e^{-kx}\cos kx + 2k^2RM_0e^{-kx}(\cos kx - \sin kx) + \left[\rho(H-x)+p\right]R,$$
$$M(x) = -\frac{1}{k}P_0e^{-kx}\sin kx + M_0e^{-kx}\cdot(\cos kx + \sin kx), \tag{1}$$
$$K(x) = \mu M^{P_0}(x) + \mu M^{M_0}(x),$$

where k – attenuation coefficient, ρ – the density of the liquid the tank is filled with, P_0 – boundary force, M_0 – edge moment.

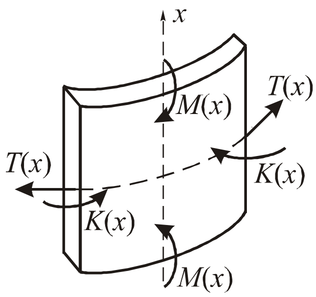

Figure 2. Internal forces in the wall of the cylindrical tank.

According to [12], the attenuation coefficient is calculated by the formula

$$k = \frac{\sqrt[4]{3\left(1-\mu^2\right)}}{\sqrt{R\delta_1}},$$

where μ – Poisson's ratio of the tank material.

The directions of the boundary forces in the wall of the cylindrical tank and its bottom are shown in Figure 3.

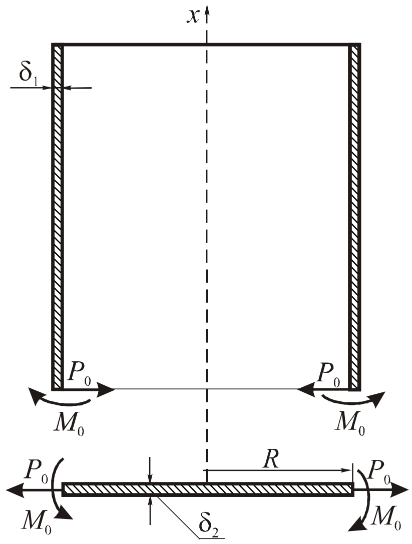

Figure 3. Boundary forces in the wall of the cylindrical tank and on the round bottom.

Research results To determine the boundary forces, let us write the equation of deformations compatibility of the tank wall with the bottom:

$$\begin{cases} \Delta_1 = \Delta_2; \\ \vartheta_1 = \vartheta_2, \end{cases} \tag{2}$$

where Δ_1, ϑ_1 – radial displacement and rotation angle of the lower edge of the wall of the cylindrical tank; Δ_2, ϑ_2 – radial displacement and rotation angle of the bottom.

Since we consider the walls of a cylindrical tank to be loaded with hydraulic q and gas pressure p (Figure 1), the expressions for radial displacements and rotation angles, taking into account the boundary force P_0 and the edge moment M_0 of the lower edge of the tank's wall and the bottom, are presented as follows

$$\Delta_1 = \Delta_1^q + \Delta_1^p + \Delta_1^{P_0} + \Delta_1^{M_0}; \quad \vartheta_1 = \vartheta_1^q + \vartheta_1^p + \vartheta_1^{P_0} + \vartheta_1^{M_0}; \tag{3}$$

$$\Delta_2 = \Delta_2^q + \Delta_2^p + \Delta_2^{P_0} + \Delta_2^{M_0}; \quad \vartheta_2 = \vartheta_2^q + \vartheta_2^p + \vartheta_2^{P_0} + \vartheta_2^{M_0}, \tag{4}$$

where Δ^q, Δ^p, Δ^{P_0}, Δ^{M_0} – radial displacements caused, respectively, by hydrostatic pressure *q*, gas pressure p, boundary force P_0 and edge moment M_0. Angles of rotation ϑ are denoted by similar indices.

The values of radial Δ and angular deformations ϑ included in expressions (3) for a cylindrical tank with a medium radius R, wall thickness δ_1 made of a material with modulus of elasticity E and Poisson's ratio μ, filled with a liquid with density ρ up to height H and additionally loaded with gas pressure p are as follows [12]:

$$\begin{aligned} &\Delta_1^p = -\frac{2-\mu}{2\delta_1 E} pR^2; \ \Delta_1^q = -\frac{\rho R^2}{\delta_1 E} H; \ \Delta_1^{P_0} = -\frac{2kR^2}{\delta_1 E} P_0; \ \Delta_1^{M_0} = -\frac{2k^2 R^2}{\delta_1 E} M_0; \\ &\vartheta_1^p = 0; \qquad \vartheta_1^q = -\frac{\rho R^2}{\delta_1 E}; \quad \vartheta_1^{P_0} = \frac{2k^2 R^2}{\delta_1 E} P_0; \ \vartheta_1^{M_0} = -\frac{4k^3 R^2}{\delta_1 E} M_0. \end{aligned} \tag{5}$$

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The bottom of the tank, which lies on a solid base, is a round plate with a thickness δ_2 and radius R (Figure 3). The ratio of the plate's radius to its thickness – R/δ_2 is a large value. According to [12], the radial displacements and rotation angles of the round plate are the following:

$$\Delta_2^q = \Delta_2^p = \Delta_2^{M_0} = 0;\ \vartheta_2^q = \vartheta_2^p = \vartheta_2^{P_0} = \vartheta_2^{M_0} = 0;\ \Delta_2^{P_0} = -\frac{1-\mu}{E}\cdot\frac{P_0 R}{\delta_2}. \tag{6}$$

In view of (3) – (6) in (2) we have:

$$\begin{cases} \Delta_1^q = \Delta_1^p = \Delta_1^{P_0} = \Delta_2^{M_0} = \Delta_2^{P_0}; \\ \vartheta_1^q = \vartheta_1^{P_0} = \vartheta_1^{M_0} = 0, \end{cases}$$

or

$$\begin{cases} -\dfrac{\rho R^2}{\delta_1 E}H - \dfrac{2-\mu}{2\delta_1 E}pR^2 + \dfrac{2kR^2}{\delta_1 E}P_0 - \dfrac{2k^2R^2}{\delta_1 E}M_0 = -\dfrac{1-\mu}{E\delta_2}P_0 R; \\ -\dfrac{\rho R^2}{\delta_1 E} + \dfrac{2k^2R^2}{\delta_1 E}P_0 - \dfrac{4k^3R^2}{\delta_1 E}M_0 = 0. \end{cases} \tag{7}$$

After the conversion, let us write the system of equations (7) as follows

$$\begin{cases} \left(2k + \dfrac{1-\mu}{R}\cdot\dfrac{\delta_1}{\delta_2}\right)P_0 - 2k^2 M_0 = \left(\rho H + \dfrac{2-\mu}{2}p\right); \\ 2k^2\cdot P_0 - 4k^3\cdot M_0 = \rho. \end{cases} \tag{8}$$

From the system of equations (8) let us determine the unknown boundary force P_0 and edge moment M_0:

$$M_0 = \frac{\rho\left(2k + \dfrac{1-\mu}{R}\cdot\dfrac{\delta_1}{\delta_2}\right) - 2k^2\left(\rho H + \dfrac{2-\mu}{2}p\right)}{4k^4 - \left(2k + \dfrac{1-\mu}{R}\cdot\dfrac{\delta_1}{\delta_2}\right)4k^3};\quad P_0 = \frac{\rho H + \dfrac{2-\mu}{2}p + 2k^2}{2k + \dfrac{1-\mu}{R}\cdot\dfrac{\delta_1}{\delta_2}}. \tag{9}$$

After taking into account expressions (9) for M_0 and P_0 in (1) we will obtain expressions for determining the internal forces: circular force $T(x)$, meridian $M(x)$ and circular $K(x)$ bending moments:

$$\begin{aligned} T(x) = &-2kR\frac{\rho H + \dfrac{2-\mu}{2}p + 2k^2}{2k + \dfrac{1-\mu}{R}\cdot\dfrac{\delta_1}{\delta_2}}e^{-kx}\cos kx + \\ &+2k^2R\frac{\rho\left(2k + \dfrac{1-\mu}{R}\cdot\dfrac{\delta_1}{\delta_2}\right) - 2k^2\left(\rho H + \dfrac{2-\mu}{2}p\right)}{4k^4 - \left(2k + \dfrac{1-\mu}{R}\cdot\dfrac{\delta_1}{\delta_2}\right)4k^3}e^{-kx}(\cos kx - \sin kx) + \left[\rho(H-x) + p\right]R; \end{aligned} \tag{10}$$

$$M(x) = -\frac{\rho H + \frac{2-\mu}{2}p + 2k^2}{k\left(2k + \frac{1-\mu}{R}\cdot\frac{\delta_1}{\delta_2}\right)} e^{-kx}\sin kx +$$
$$+\frac{\rho\left(2k + \frac{1-\mu}{R}\cdot\frac{\delta_1}{\delta_2}\right) - 2k^2\left(\rho H + \frac{2-\mu}{2}p\right)}{4k^4 - \left(2k + \frac{1-\mu}{R}\cdot\frac{\delta_1}{\delta_2}\right)4k^3} e^{-kx}(\cos kx + \sin kx);$$

$$K(x) = \mu \cdot M^{P_0}(x) + \mu \cdot M^{M_0}(x) = \mu \cdot M_0 .$$

Taking into account (10) for internal forces, we will calculate normal stresses σ_M and σ_T in the wall of the cylindrical tank, arising from the action of boundary forces, by the formula:

$$\sigma_M = \frac{6\cdot|M(X)|}{\delta_1^2}; \quad \sigma_T = \frac{T(x)}{\delta_1} + \frac{6\cdot|M(X)|}{\delta_1^2}. \tag{11}$$

According to [4, 12], the forces and stresses associated with the edge effect at a distance $x = \pi / k$ constitute only a few percent of their values at the edge of the shell, so it is not reasonable to take them into account at a further distance.

The influence of boundary forces and additional gas pressure on the magnitude of stresses will be considered based on the example of a steel $(\mu = 0.3)$ thin-walled tank with dimensions $R = 7.5\ m,\ H = 9\ m,\ \delta_1 = \delta_1 = 10^{-2}\ m$ filled with liquid (oil) with a density of $\rho = 760\ kg/m^3$. The curves of changes in stresses σ_M and σ_T obtained by formulas (11) taking into account (10) are shown in Figure 4. Curves 1 in Figure 4 show the change of stresses in the tank wall under additional exposure to gas pressure $p = 93.3\ kPa$, curves 2 – pressure $p = 186.6\ kPa$. Under the combined effect of gas and hydraulic pressure the stress σ_M acquires the largest value in the wall of the tank near the joint with the bottom: when $x = 0$, then $\sigma_M = 126\ MPa$ with gas pressure $p = 93.3\ kPa$, (Figure 4, a, curve 1). Double increase of the additional gas pressure (Figure 4, a, curve 2) also causes a double increase in stress σ_M from the boundary forces.

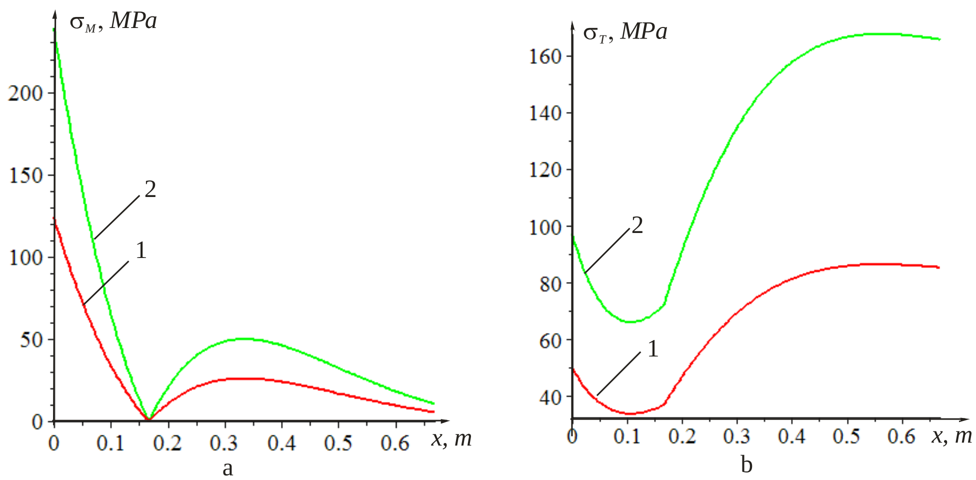

Figure 4. Graphs of changes in stresses from boundary forces in the wall of a cylindrical tank.

The stress σ_T in the tank wall near the joint with the bottom, when $x=0$, is equal to $\sigma_T=50$ *MPa* at additional gas pressure $p=93.3$ *kPa*, (curve 1). When the pressure doubles, they are increased almost twice as well. (Figure 4, b, curve 2). These stresses reach the greatest value at a distance of $x=0.5$ *m* from the flat bottom of the tank. In a cylindrical tank with a spherical bottom [12], the largest circular stresses occur at a distance $x=1.447\sqrt{R\cdot\delta_1}$. With the specified sizes of the tank in case of a spherical bottom the value x constitutes 0.396 *m*.

According to the momentless theory of shell strength, provided that the cylindrical tank is exposed only to hydrostatic pressure of the liquid, the values of the membrane stresses in the wall are much smaller. Membrane stresses are equal to: meridian – $\sigma_m=38$ *MPa*, circular $-\sigma_\theta=76$ *MPa*. The meridian membrane stress at a given tank load is 3.6 times less than the stress $\sigma_M=126$ *MPa* calculated taking into account the boundary forces at the tank wall-bottom joint.

Conclusions

The strength of a thin-walled cylindrical tank was evaluated under the combined action of the hydrostatic pressure of the liquid and the additional gas pressure. To determine the stresses in the dangerous intersection near the flat bottom of the tank, the presence of boundary forces is taken into account. The dependences for determining the boundary forces were obtained by solving the equation of deformations compatibility of a thin-walled cylindrical shell and a round plate resting on a solid surface.

It was found that the meridian stresses in the tank wall at the junction with the round bottom plate are 3.6 times greater than the values of such stresses, determined by the momentless theory of shells. Therefore, taking into account additional gas pressure and boundary forces to calculate the strength of a cylindrical tank with a flat bottom increases the values of stresses and changes the correlations between them. Such factors of impact on the strength must be taken into account both during the design and during the use of the tanks.

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